STAT/MATH 416 Spring 09
Practice Midterm #2
April 9, 2009

You are not allowed to use books or notes. Calculators approved for the actuarial exams are permitted. The last page contains a table for the cumulative distribution function of a standard normal variable. Please read the directions carefully. For the free response problems, show your work. The maximum obtainable score is 195. The exam will graded out of 175 points, so you may earn extra credit. You have 1 hour to complete it. Do not detach any pages. If you ran out of allocated space, use the backs of the pages for calculations.

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<td>2</td>
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1. Please clearly mark only one answer for each of the five multiple choice questions. If the exact answer is not available, choose the closest numerical answer.

(a) (15 points) A moving company is also providing insurance for its cargo. In cases when there is any damage to the shipment, the dollar amount of damage per shipment is exponentially distributed with an average of 200. If the deductible for the insurance policy is $250 (i.e., only claims over $250 are submitted), what is the expected amount of a submitted claim?

   A 129  B 250  C 317  D 450  E 567

(b) (15 points) The number of lightning hits that strike within one mile of campus in one year is a Poisson distributed random variable. Assume that the probability of exactly 2 hits is the same as the probability of exactly 3 hits. What is the standard deviation of $X$?

   A 1.41  B 1.50  C 1.73  D 2.00  E 3.00
(c) (15 points) A probability density function for a variable $X$ is given by

$$f(x) = \begin{cases} \frac{1}{4}xe^{-\frac{x}{2}} & : x \geq 0, \\ 0 & : \text{otherwise}. \end{cases}$$

Let $Y = \frac{1}{4}X - 3$. The mean $E[Y]$ and the variance $\text{Var}(Y)$ of $Y$ are

$$A \quad -3, \quad 0.25 \quad B \quad -3, \quad 0.0625 \quad C \quad -2.5, \quad 0.25 \quad D \quad 4, \quad 8 \quad E \quad -2, \quad 0.5$$

(d) (15 points) What is the smallest number of flips of a fair coin needed to have the probability of at most 0.01 of not a single heads?

$$A \quad 6 \quad B \quad 7 \quad C \quad 8 \quad D \quad 9 \quad E \quad 10$$
(e) (15 points) A joint probability density function for random variables $X$ and $Y$ is given by

$$f_{XY}(x, y) = \begin{cases} \frac{1}{2\sqrt{2\pi}}xe^{-\frac{1}{4}x(x+y)} & : 0 \leq x \leq y, \\ 0 & : \text{otherwise.} \end{cases}$$

Then the marginal density $f_X(x)$ is

- $A \ e^{-x}, \ x \geq 0, \ 0 \text{ otherwise}$
- $B \ \frac{1}{4}e^{-\frac{1}{2}x}, \ x \geq 0, \ 0 \text{ otherwise}$
- $C \ \frac{1}{2}e^{-\frac{1}{2}x}, \ x \geq 0, \ 0 \text{ otherwise}$
- $D \ \frac{1}{\sqrt{\pi}}e^{-\frac{1}{4}x^2}, \ x \geq 0, \ 0 \text{ otherwise}$
- $E \ \frac{2}{\sqrt{2\pi}}e^{-\frac{1}{2}x^2}, \ x \geq 0, \ 0 \text{ otherwise}$
2. (a) (15 points) A card is drawn from a full deck of cards at random with replacement 1000 times. What is the probability that suite ♠ appears at least 265 times?

(b) (15 points) If this experiment is repeated again, what is the probability that the number of ♠ draws from the first experiment is more than 10 larger than the number of ♠ draws from the second experiment?
3. (20 points) A scanner maker receives sensor shipments from two suppliers, A and B, with 70% of shipments coming from A. Each part coming from A has 0.005 probability of being defective while each part from B has 0.01 probability of a defect. A shipment of 1000 sensors has been received from one of the manufacturers, and after an inspection, exactly 10 of them were found defective. What is the probability that the shipment came from B?
4. (a) (20 points) A random variable $X$ has a probability density function

$$f_X(x) = \begin{cases} 
  a + x + bx^2 & : x \in (0,1), \\
  0 & : \text{otherwise}.
\end{cases}$$

Find $a$ and $b$ given that $E[X] = \frac{5}{8}$.

(b) (10 points) Find $E[\sqrt{X}]$. 
5. (20 points) \( X \) is a random variable with a probability density function

\[ f_X(x) = \frac{15}{4} x^2 (1 - x^2) , \text{ } x \in (0, 1) \text{ and } 0 \text{ otherwise.} \]

Find the probability density function for \( Y = X^2 \).
6. (20 points, extra credit, attempt once the rest of the problems are completed) Suppose $X$ and $Y$ are standard uniform random variables. Find the distribution function for $X/Y$. 