Please show all your work, don’t just write down the final number.

- Reading: 11.2-3, 8.2, 9.3, handout on convolution, handout on expectation of the sum of random variables
- Exercises 11.2.3, 11.2.5
- Ross (8th edition, problems may or may not match the 7th edition)

6.56 If $X$ and $Y$ are independent and identically distributed uniform random variables on $(0, 1)$, compute the joint density of
(a) $U = X + Y$, $V = X/Y$;
(b) $U = X$, $V = X/Y$;
(c) $U = X + Y$, $V = X/(X + Y)$.

6.57 Repeat Problem 6.56 when $X$ and $Y$ are independent exponential random variables, each with parameter $\lambda = 1$.

6.58 If $X_1$ and $X_2$ are independent exponential random variables, each having parameter $\lambda$, find the joint density function of $Y_1 = X_1 + X_2$ and $Y_2 = e^{X_1}$.

7.6 A fair die is rolled 10 times. Calculate the expected sum of the 10 rolls.

7.7 Suppose that $A$ and $B$ each randomly and independently choose 3 of 10 objects. Find the expected number of objects
(a) chosen by both $A$ and $B$;
(b) not chosen by either $A$ or $B$;
(c) chosen by exactly one of $A$ and $B$.

7.13 A set of 1000 cards numbered 1 through 1000 is randomly distributed among 1000 people with each receiving one card. Compute the expected number of cards that are given to people whose age matches the number on the card.

7.21 (Difficult) For a group of 100 people, compute
(a) the expected number of days of the year that are birthdays of exactly 3 people;
(b) the expected number of distinct birthdays.

7.30 If $X$ and $Y$ are independent and identically distributed with mean $\mu$ and variance $\sigma^2$, find $E\left[(X - Y)^2\right]$.

7.33 If $E[X] = 1$ and $Var(X) = 5$, find
(a) $E\left[(2 + X)^2\right]$;
(b) $Var(4 + 3X)$.

7.36 Let $X$ be the number of 1’s and $Y$ the number of 2’s that occur in $n$ rolls of a fair die. Compute $Cov(X, Y)$. 

1
7.38 The random variables $X$ and $Y$ have a joint density function given by

$$f_{XY}(x,y) = \begin{cases} 2e^{-2x/y} & 0 \leq x < \infty, \ 0 \leq y \leq x \\ 0 & \text{otherwise} \end{cases}$$

compute $Cov(X,Y)$.

7.40 The joint density function of $X$ and $Y$ is given by

$$f_{XY}(x,y) = \frac{1}{y} e^{-(y+x/y)}, \ x > 0, \ y > 0$$

Find $E[X]$, $E[Y]$, and show that $Cov(X,Y) = 1$. 