| Name | SOLUTION |
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| PID # | |
| Instructor | Sergey Kirshner |

# STAT/MATH 416 001 Fall 2009
## Practice Quiz #3
November 2, 2009

You are not allowed to use books or notes. Non-programmable calculators approved for 1/P exam are permitted. Please read the directions carefully. The quiz is graded out of 50 points. You have 20 minutes to complete it. Use the normal distribution table on the last page if needed. Please show all your work.

1. (10 points) Assume that the number of policies sold in a day by a small insurance company is Poisson distributed. The probability that the company would sell any policies today is 0.7769. What is the probability that the company would sell at least two policies today?

   **Solution:** Let $X$ be the random variable indicating the number of policies sold today. $X$ is Poisson distributed, so

$$
\begin{aligned}
P(X=i) &= \frac{\lambda^i}{i!}e^{-\lambda},\ i=0,1,\ldots. \\
0.7769 &= P(X\geq 1) = 1-P(X=0) = 1-\frac{\lambda^0}{0!}e^{-\lambda} \Longrightarrow \lambda = -\ln(1-0.7769)\approx 1.5; \\
P(X\geq 2) &= 1-P(X=0)-P(X=1) = 1-\frac{\lambda^0}{0!}e^{-\lambda}-\frac{\lambda^1}{1!}e^{-\lambda} = 1-e^{-\lambda}(1+\lambda)\approx \mathbf{0.4422}.
\end{aligned}
$$

2. (15 points) The life of a device is exponentially distributed, with a device lasting on average 2000 hours. A device has been in operation for 1500 hours. What is the probability that it will last at least another 500 hours?

   **Solution:** Let $X$ denote the lifespan of the device in hours. Then $E[X]=2000$, so the parameter $\lambda=\frac{1}{2000}$, and $F(X)=1-e^{-\frac{1}{2000}x}$.

$$
\begin{aligned}
P(X>1500+500|X>1500) &= \frac{P(X>1500, X>2000)}{P(X>1500)} = \frac{P(X>2000)}{P(X>1500)} \\
&= \frac{1-\left(1-e^{-\frac{2000}{2000}}\right)}{1-\left(1-e^{-\frac{1500}{2000}}\right)} = \frac{e^{-1}}{e^{-0.75}} = e^{-0.25}\approx \mathbf{0.7788}.
\end{aligned}
$$

   Alternatively, an exponentially distributed variable is memoryless, i.e., $P(X>s+t|X>t)=P(X>s)$, so

$$P(X>1500+500|X>1500) = P(X>500) = e^{-\frac{500}{2000}},$$

   simplifying the derivation significantly.

3. (15 points) An urn contains 40 white balls and 60 black balls. A ball is drawn with replacement 100 times. What is the probability that a white ball appeared at least 45 times? (Use continuity correction)

**Solution:** Let $X$ denote the number of white balls from the 100 draws.
$X \sim Bin\,(n = 100, p = 0.4)$.

$$\begin{aligned} P\,(X \geq 45) &= P\,(44.5 < X < 100.5) = P\left(\frac{44.5 - 100 \times 0.4}{\sqrt{100 \times 0.4 \times 0.6}} < X < \frac{100.5 - 100 \times 0.4}{\sqrt{100 \times 0.4 \times 0.6}}\right) \\ &\approx \Phi\,(12.3495) - \Phi\,(0.9186) = \mathbf{0.1792}. \end{aligned}$$

4. (10 points) If $X$ is a normal variable with mean 2 and variance 9, find the probability density function for $Y = -X^3$.

**Solution:** $Y = g\,(X) = -X^3$. Thus $X = g^{-1}\,(Y) = -\sqrt[3]{Y}$. The probability density function for $X$ is

$$f_X\,(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} = \frac{1}{3\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-2}{3}\right)^2}.$$

Then

$$f_Y\,(y) = f_X\left(g^{-1}\,(Y)\right)\left|\frac{d}{dy} g^{-1}\,(y)\right| = \frac{1}{3\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{-\sqrt[3]{y}-2}{3}\right)^2} \times \frac{1}{3}\,|y|^{-\frac{2}{3}}.$$

The domain for $f_Y$ is the range of the transformation function $g$. In this case, the range is $\mathbb{R}$, so $y \in \mathbb{R}$.