Name	SOLUTION
PID #	
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STAT/MATH 416 001 Fall 2009
Practice Quiz #2
October 9, 2009

You are not allowed to use books or notes. Non-programmable calculators approved for 1/P exam are permitted. Please read the directions carefully. The quiz is graded out of 50 points. You have 20 minutes to complete it. Please show all your work.

1. (20 points) An assembly plant receives the same number of part shipments from two suppliers. Each part from the supplier A has 2% chance of being defective, but chances of a defect are only 1% if the part comes from supplier B. A randomly selected shipment is opened and checked, and out of 50 parts, 1 is defective. What is the probability that this shipment is from A?

   Solution: Let event A denote that the supplier is A, $B = A^c$ that the supplier is B, and let D be the event that in a shipment, in a randomly chosen 50 parts, 1 is defective. We are asked to find

$$P(A|D) = \frac{P(A)P(D|A)}{P(D)} = \frac{P(A)P(D|A)}{P(A)P(D|A) + P(B)P(D|B)}.$$

   Given the supplier, the events of each part being defective are independent and have the same probabilities. Thus a random variable indicating the number of defective parts in a sample of a fixed size from a given supplier is distributed according to the binomial distribution. For supplier A, the probability of a defective part is $p = 0.02$, for supplier B, it is $p = 0.01$. Thus

$$\begin{aligned} P(A|D) &= \frac{P(A)P(D|A)}{P(A)P(D|A) + P(B)P(D|B)} = \frac{\frac{1}{2}\binom{50}{1}0.02^1 \times 0.98^{49}}{\frac{1}{2}\binom{50}{1}0.02^1 \times 0.98^{49} + \frac{1}{2}\binom{50}{1}0.01^1 \times 0.99^{49}} \\ &= \frac{0.02 \times 0.98^{49}}{0.02 \times 0.98^{49} + 0.01 \times 0.99^{49}} \approx \mathbf{0.5488}. \end{aligned}$$

2. (20 points) A biased coin A needs to be flipped on average 2.5 times to show heads, and a biased coin B needs on average 3 flips to show heads. Both of these coins are flipped at the same time. What is the probability of at least one of them showing heads? (Hint: first find the probabilities of landing heads for each of the coins.)

   Solution: Let X be a biased coin, and p be its probability for showing heads on a single flip. An average number of flips needed for the coin to show head is the expected value for a variable indicating the first time coin shows heads in a Bernoulli trial, i.e., a geometric random variable. The expected value of a geometric random variable with a parameter p is $\frac{1}{p}$, so for coin A, $P(\text{heads}_A) = \frac{1}{2.5} = \frac{2}{5}$, and for coin B, $P(\text{heads}_B) = \frac{1}{3}$. The event of coins A and B showing at least one heads is the complement of the event of both of them showing tails.

$$\begin{aligned} & P(\text{at least one of A and B showing heads}) \\ = \quad & 1 - P(\text{tails}_A, \text{tails}_B) = 1 - P(\text{tails}_A)\, P(\text{tails}_B) = 1 - \frac{3}{5} \times \frac{2}{3} = \mathbf{0.6}. \end{aligned}$$

3. (10 points) In an event of an accident, an insurance company pays \$40 per day for up to 5 days of a replacement car rental. Let X be the number of days for the car rental. X be a discrete random variable with the following probability mass function:

$$P(X = n) = \begin{cases} 0.5 & : n = 1, \\ 0.25 & : n = 2 \\ 0.1 & : n = 3, \\ 0.05 & : n = 4, \\ 0.1 & : n = 5, \\ 0 & : \text{otherwise.} \end{cases}$$

(a) (5 points) Compute the expected payment for a car rental under this policy.

Solution: Let Y be a random variable denoting the payment for a car rental; then $Y = 40X$.

$$\begin{aligned} E[X] &= 0.5 \times 1 + 0.25 \times 2 + 0.1 \times 3 + 0.05 \times 4 + 0.1 \times 5 = 2; \\ E[Y] &= E[40X] = 40E[X] = \mathbf{80}. \end{aligned}$$

(b) (5 points) Compute the variance of a payment for a car rental under this policy.

Solution:

$$\begin{aligned} \text{Var}(X) &= E\left[X^2\right] - (E[X])^2 \\ &= 0.5 \times 1 + 0.25 \times 4 + 0.1 \times 9 + 0.05 \times 16 + 0.1 \times 25 - 2^2 = 1.7; \\ \text{Var}(Y) &= \text{Var}(40X) = 40^2 \times \text{Var}(X) = \mathbf{2720}. \end{aligned}$$