Name	SOLUTION
PID #	

STAT/MATH 416 001 Fall 2009
Quiz #1
September 9, 2009

You are not allowed to use books or notes. Non-programmable calculators approved for 1/P exam are permitted. Please read the directions carefully. The quiz is graded out of 50 points. You have 20 minutes to complete it. Please show all your work.

1. A florist has a half dozen (6) of white and a half dozen of red roses. These flowers are used to assemble bouquets.

   (a) (5pts) How many of all bouquets have exactly 5 roses?

   Solution: Let B denote the event that a bouquet contains exactly 5 roses. The number of ways to choose 5 flowers from 12 is $|B| = \binom{\mathbf{12}}{\mathbf{5}} = \frac{\mathbf{12!}}{\mathbf{5!7!}} = \mathbf{792}$.

   (b) (5pts) How many bouquets contain exactly 3 red roses and 2 white roses?

   Solution: Let C denote the event that a bouquet contains exactly 3 red and 2 white roses. There $\binom{6}{3}$ ways to pick red roses for the bouquet and $\binom{6}{2}$ ways to pick white roses for the bouquet. Since the number of selections of white roses is the same for any selection of red roses, by the Fundamental Principle of Counting, the total number of bouquets is $|C| = \binom{\mathbf{6}}{\mathbf{3}} \times \binom{\mathbf{6}}{\mathbf{2}} = \mathbf{300}$.

   (c) (5pts) How many bouquets of 5 roses contain at least one red rose?

   Solution: Let D denote the event that a 5-rose bouquet contains at least one red rose. Each bouquet of 5 roses (B) either contains no red roses (denote this event by D_1) or at least 1 red rose (D), but not both, so $D \cup D_1 = B, D \cap D_1 = \emptyset$. Thus

$$|D| = |B| - |D_1| = \binom{\mathbf{12}}{\mathbf{5}} - \binom{\mathbf{6}}{\mathbf{5}} = \mathbf{792 - 6 = 786}.$$

   Alternatively, a bouquet of 5 roses contains at least one red rose if and only if it contains 1 and 4, 2 and 3, 3 and 2, 4 and 1, and 5 and 0, red and white roses, respectively. Thus

$$|D| = \binom{6}{1} \times \binom{6}{4} + \binom{6}{2} \times \binom{6}{3} + \binom{6}{3} \times \binom{6}{2} + \binom{6}{4} \times \binom{6}{1} + \binom{6}{5} \times \binom{6}{0} = 786.$$

2. (10pts) An urn contains 3 red balls and 6 blue balls. A second urn contains 5 red balls and an unknown number of blue balls. A single ball is drawn from each urn. The probability that both balls are of the same color is $\frac{19}{36}$. Calculate the number of blue balls in the second urn.

Solution: Let x denote the number of blue balls in the second urn.

$$\begin{aligned}\frac{19}{36} &= P\,(\text{both balls are the same color}) = P\,(\text{both balls are red}) + P\,(\text{both balls are blue}) \\ &= \frac{3}{9} \times \frac{5}{x+5} + \frac{6}{9} \times \frac{x}{x+5} = \frac{5+2x}{3x+15};\end{aligned}$$

$$\begin{aligned}19\,(x+5) &= 12\,(5+2x)\,; \\ 5x &= 35; \\ x &= \mathbf{7}.\end{aligned}$$

3. (15pt) A total of 35 students are taking probability course. 20 are majoring in mathematics, 15 in statistics, 10 in finance, 7 majoring in mathematics and statistics, 5 majoring in mathematics and finance, 4 majoring in statistics and finance, and 1 majoring in all three. What is the number of people *not* majoring in either mathematics, statistics, or finance?

Solution: Let S denote the set of all students in the course, M denote a set of math majors, S denote a set of stat majors, and F denote a set of finance majors. Using the Inclusion-Exclusion Principle, we find that

$$\begin{aligned}|(M \cup S \cup F)^c| &= |S| - |M \cup S \cup F| \\ &= |S| - (|M| + |S| + |F| - |M \cap S| - |M \cap F| - |S \cap F| + |M \cap S \cap F|) \\ &= 35 - (20 + 15 + 10 - 7 - 5 - 4 + 1) = 35 - 30 = \mathbf{5}.\end{aligned}$$

4. (10pts) Let X be a random variable denoting the number of aces in a randomly drawn 5-card poker hand. What is the probability mass function for X?

Solution: All of 5-card hands are assumed to be equiprobable, so if i denotes the number of aces, $P(X = i) = \frac{\text{\# of hands with } i \text{ aces}}{\text{total \# of hands}}$. There are $\binom{52}{5} = \frac{52!}{5!47!}$ different ways to choose a 5-card hand. There are only 4 aces in the deck, so we need to find $P(X = i)$ for $i \in \{0, 1, 2, 3, 4\}$ as $P(X = i) = 0$ otherwise. The number of ways to choose a hand with exactly i aces is equivalent to choosing i aces out of 4 and $5 - i$ non-aces from 48 non-ace cards, $\binom{4}{i}\binom{48}{5-i}$. Thus

$$p_X(i) = \begin{cases} \frac{\binom{4}{0}\binom{48}{5}}{\binom{52}{5}} \approx \mathbf{0.6588} & i = 0, \\ \frac{\binom{4}{1}\binom{48}{4}}{\binom{52}{5}} \approx \mathbf{0.2995} & i = 1, \\ \frac{\binom{4}{2}\binom{48}{3}}{\binom{52}{5}} \approx \mathbf{0.0399} & i = 2, \\ \frac{\binom{4}{3}\binom{48}{2}}{\binom{52}{5}} \approx \mathbf{0.0017} & i = 3, \\ \frac{\binom{4}{4}\binom{48}{1}}{\binom{52}{5}} \approx \mathbf{0.0002} & i = 4, \\ 0 & \text{otherwise.} \end{cases}$$

For the test, it would be acceptable to leave the answer in the form of the ratio of the binomial coefficients (unless otherwise stated).