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Student ID #	
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STAT/MATH 416 Fall 09

Practice Midterm #1

September 28, 2009

You are not allowed to use books or notes. Non-programmable calculators approved for 1/P exam are permitted. Please read the directions carefully. The exam is graded out of 175 points with a possibility of additional 20 points for solving an extra credit problem. You have 50 minutes to complete the exam. **Please show all your work.**

Problem #	Number of points
1	/20
2	/20
3	/20
4	/20
5	/30
6	/35
7	/30
8 (extra)	/20
Total	/175

1. (20 points) A fair coin is flipped 10 times. What is the probability that more than two tosses are heads?

2. (20 points) A student is taking midterms in Linear Algebra, Probability, and Analysis. The probabilities of getting an A in Linear Algebra, Probability, and Analysis are 0.3, 0.5, and 0.25, respectively. Given that grades on the midterms in these courses are mutually independent, the probability that a student will get at least one A.

3. (20 points) 60% of students are well prepared for the exam, 40% are poorly prepared. A well-prepared student has 0.6 probability of receiving an A on the exam, 0.3 of receiving a B, and 0.1 of receiving a grade less than B. A poorly prepared student has 0.2 probability of receiving an A, 0.2 probability of receiving a B, 0.6 probability of receiving a grade below B. Find the probability that a student was well prepared given that he/she received a B on the exam.

4. (20 points) In a particular zoo, 20 animals have fur, 25 are carnivorous, and 8 are furry carnivores with natural habitat in Africa. If 35 of zoo's occupants are either carnivorous or furry, how many of its furry carnivores are not naturally found in Africa?

5. Die A has one yellow and five orange sides, and die B has one orange and five yellow sides. A fair coin is flipped, and die A is selected if it lands heads, and die B is selected if it lands tails. Assume that the selected die was rolled once, and it rolled orange side up.

(a) (15 points) What is the probability that die A is selected?

(b) (15 points) If the same die is rolled again, what is the probability that the orange side will come up again?

6. A random variable X has $\mu = E[X] = 0.75$ and $\sigma = SD(X) = 0.5$.

(a) (5 points) What can be said about $P(0 < X < 1.5)$?

(b) (15 points) Now, suppose you are told that the functional form for a p.d.f. for X is given as

$$f_X(x) = \begin{cases} \frac{3}{c}(-ax^2 - 6x + 34) & : x \in (0, 2), \\ 0 & : \text{otherwise.} \end{cases}$$

Find a and c . (Hint: you don't really need both $E[X]$ and $SD(X)$; only $E[X]$ would suffice.)

(c) (10 points) Using function in (ii), find distribution function for X , $F_X(x)$. Leave the expression in terms of fractions, but be sure to specify it for *all* values in \mathbb{R} .

(d) (5 points) Find what quantile correspond to $x = 1$.

7. Suppose that a probability that a game of baseball lasts N innings is

$$P(N = n) = \begin{cases} a \times \left(\frac{1}{3}\right)^{n-9}, & : n \geq 9 \\ 0 & : \text{otherwise.} \end{cases}$$

(a) (10 points) Find a .

(b) (5 points) Find the mode of X .

(c) (15 points) What is the probability that a game ends in less than 12 innings if it went into extra-innings (i.e., $N > 9$)?

8. (20 points, extra credit) A fair die is rolled twice. Compute the probability mass function, the expectation and the variance for the random variable X equal to the minimum value to appear on the two rolls.