- Problem 6.4 from the textbook. Feel free to use R to analyze this dataset, if you prefer. Keep in mind that .tsm data files from the ITSM are just ASCII text files. You can easily open them using Notepad or WordPerfect, save as a .txt file and then import into R.

- For an ARCH(1) process $X_t = \sigma_t \varepsilon_t$ with $\sigma_t^2 = \alpha_0 + \alpha_1 X_{t-1}^2$ verify that some of my in-class statements are true. In particular,

  1. Check that
  $$E(\sigma_t^4) = \frac{\alpha_0^2}{1-\alpha_1}\frac{1+\alpha_1}{1-3\alpha_1^2}$$
  2. Deduce that
  $$E(X_t^4) = 3\left(\frac{1-\alpha_1^2}{1-3\alpha_1^2}\right)$$

- Show that for an IGARCH (1,1) model, for $j \geq 0$
$$E(\sigma_{t+j}^2|\mathcal{F}_{t-1}) = j\alpha_0 + \sigma_t^2$$
By showing this, you are, in fact, demonstrating that IGARCH model generates **persistent** volatility

- Consider a GARCH (2,3) model $X_t = \sigma_t \varepsilon_t$ where $\sigma_t^2 = \alpha_0 + \sum_{i=1}^{2} \beta_i \sigma_{t-i}^2 + \sum_{j=1}^{3} \alpha_j X_{t-j}^2$ Show that X_t^2 can be written as an $ARMA(3,2)$ model in terms of the process $\nu_t = \sigma_t^2(\varepsilon_t^2 - 1)$. Identify the parameters of this process in terms of the parameters of the original parameters of the GARCH(2,3) model.