- Suppose the process $\{X_t\}$ is stationary and has acvf $\gamma_X(h)$. A new process $\{Y_t\}$ is defined by $Y_t = X_t - X_{t-1}$. Obtain the acvf of $\{Y_t\}$ in terms of $\gamma_X(h)$ and find $\gamma_Y(h)$ when $\gamma_X(h) = \lambda^{|h|}$

- For an AR(2) process show

$$X_t - \alpha_1 X_{t-1} - \alpha_2 X_{t-2} = \epsilon_t$$

show that its stationarity region is

$$\begin{aligned} \alpha_1 + \alpha_2 &< 1 \\ \alpha_1 - \alpha_2 &> -1 \\ |\alpha_2| &< 1 \end{aligned}$$

Note that if you plot this region, you'll obtain a triangular shape! For $\alpha_1 = \frac{1}{3}$ and $\alpha_2 = \frac{2}{9}$, show that the acf of X_t is given by

$$\rho(h) = \frac{16}{21}\left(\frac{2}{3}\right)^{|h|} + \frac{5}{21}\left(-\frac{1}{3}\right)^{|h|}$$

for $h = 0, \pm 1, \pm 2, \ldots$.

- Show that the acf of the second order MA process

$$X_t = Z_t + 0.7Z_{t-1} - 0.2Z_{t-2}$$

is given by

$$\begin{aligned} \rho(h) &= 1\,, h = 0 \\ \rho(h) &= 0.37\,, h = \pm 1 \\ &-0.13\,, h = \pm 2 \\ &0\,, \text{ otherwise} \end{aligned}$$

- Suppose we have an AR(1) process with mean μ, defined as

$$X_t - \mu = \alpha_1(X_{t-1} - \mu) + Z_t$$

Let $\bar{x}_{(1)}$ and $\bar{x}_{(2)}$ be the means of the first and last $n-1$ observations. Show that the least squares estimates are:

$$\hat{\mu} = \frac{\bar{x}_{(2)} - \hat{\alpha}_1 \bar{x}_{(1)}}{1 - \hat{\alpha}_1}$$

and

$$\hat{\alpha}_1 = \frac{\sum_{t=1}^{n-1}(x_t - \hat{\mu})(x_{t+1} - \hat{\mu})}{\sum_{t=1}^{n-1}(x_t - \hat{\mu})^2}$$

Deduce that, if we ignore the difference between $\bar{x}_{(1)}$ and $\bar{x}_{(2)}$, the above two formulas become

$$\hat{\mu} = \bar{x}$$

and

$$\hat{\alpha}_1 = \frac{\sum_{t=1}^{n-1}(x_t - \bar{x})(x_{t+1} - \bar{x})}{\sum_{t=1}^{n-1}(x_t - \bar{x})^2}$$

- Consider the monthly U.S. unemployment rate from January 1951 to February 2004 in the file m-unemhelp.txt. The data are seasonally adjusted and obtained from the Federal Reserve Bank in St. Louis. Build a time series model for the series and use the model to forecast the unemployment rate for March, April, and May 2004. In addition, compute the average period of business cycles if they exist. (Note that more than one model fits the data well. You only need an *adequate* model.)