STAT 516 HW4

1.7.6
Part (a)
\[ P(\mid X \mid 1) = \int_{-1}^{1} x^2 / 18 \, dx = 1/27 \]
\[ P(\mid X \mid 3) = \int_{-3}^{3} x^2 / 18 \, dx = 1 \]
Part (b)
\[ P(\mid X \mid 1) = \int_{-1}^{1} (x + 2) / 18 \, dx = 2/9 \]
\[ P(\mid X \mid 3) = \int_{-2}^{3} (x + 2) / 18 \, dx = 25/36 \]

1.7.8
(a) 
X=1
(b) 
Calculate derivative \( 12(2x - 3x^3) = 0 \)
X=2/3
(c) 
Calculate derivative \( xe^{-x}(1 - x/2) \)
So X=2

1.7.10
\[ \int_{0}^{\xi_{0.2}} 4x^3 \, dx = 0.2 : \]

hence, \( \xi_{0.2}^4 = 0.2 \) and \( \xi_{0.2} = 0.2^{1/4} \).

1.7.15
\[ P(X > z) = 1 - F_X(z) \]
\[ P(Y > z) = 1 - F_Y(z) \]
So, the conclusion is correct.

1.7.16 Since \( \Delta > 0 \)
\[ X > z \Rightarrow Y = X + \Delta > z. \]
Hence, \( P(X > z) \leq P(Y > z) \).
1.8.3

\[ EX = 3 \]
\[ EX^2 = 11 \]

\[ E(X + 2)^2 = EX^2 + 4EX + 4 = 11 + 12 + 4 = 27 \]

1.8.5

(a)

\[ EX^2 = P(0) \cdot 0 + (1 - P(0)) \cdot 1 = \frac{3}{4} \]

(b)

\[ EX = P(0) \cdot 0 + P(1) \cdot 1 - P(-1) = P(1) - P(-1) = \frac{1}{4} \]

So

\[ P(1) + P(-1) = \frac{3}{4} \]
\[ P(1) = \frac{1}{2} \]
\[ P(-1) = \frac{1}{4} \]

1.8.10 The expectation of \( X \) does not exist because

\[ E(|X|) = \frac{2}{\pi} \int_0^\infty \frac{x}{1 + x^2} \, dx = \frac{1}{\pi} \int_1^\infty \frac{1}{u} \, du = \infty, \]

where the change of variable \( u = 1 + x^2 \) was used.

\( f(x) \) is symmetric about 0.