So the statement is correct for 80% CI.

4.5.1

\[ \gamma(u) = \Phi(-z_\alpha - \frac{\sqrt{n}u_0}{\sigma} + \frac{\sqrt{n}u}{\sigma}) \]

So when \( u \) increases, \( \gamma(u) \) increases too, strictly.

\( \gamma(u) \) is maximized by \( u_0 \).

Reject region:

\[ R = \bar{X} \in [u_0 + \frac{z_\alpha \sigma}{\sqrt{n}}, \infty) \]

So,

\[ \text{MAX}\{P(R | H_0)\} = P((\bar{X} - u_0) / \frac{\sigma}{\sqrt{n}} \geq z_\alpha) = \alpha \]

4.5.3 For a general \( \theta \) the probability of rejecting \( H_0 \) is

\[ \gamma(\theta) = \int_{3/4}^{1} \int_{3/4x_1}^{1} \theta^2 (x_1x_2)^{\theta - 1} \, dx_2 \, dx_1 = 1 - \left(\frac{3}{4}\right)^\theta + \theta \left(\frac{3}{4}\right)^\theta \log \left(\frac{3}{4}\right) \]

\( \gamma(1) \) is the significance level and \( \gamma(2) \) is the power when \( \theta = 2 \).
\[ \gamma(\theta) = P(\bar{X} \geq c; \theta) = P\left( \frac{\bar{X} - \theta}{\frac{5000}{\sqrt{n}}} \geq \frac{c - \theta}{\frac{5000}{\sqrt{n}}}; \theta \right) = 1 - \Phi\left( \frac{c - \theta}{\frac{5000}{\sqrt{n}}} \right). \]

Thus, solve for \( n \) and \( c \) knowing that
\[ \frac{c - 30000}{5000/\sqrt{n}} = 2.325 \quad \text{and} \quad \frac{c - 35000}{5000/\sqrt{n}} = -2.05. \]

\[ \gamma(p) = P(Y \geq c; p) = P\left( \frac{Y - np}{\sqrt{np(1-p)}} \geq \frac{c - np}{\sqrt{np(1-p)}}; p \right) \approx 1 - \Phi\left( \frac{c - np}{\sqrt{np(1-p)}} \right). \]

So solve for \( n \) and \( c \) knowing that approximately
\[ \frac{c - n(1/2)}{\sqrt{n(1/2)(1/2)}} = 1.282, \quad \frac{c - n(2/3)}{\sqrt{n(2/3)(1/3)}} = -1.645. \]

4.6.2 Suppose \( \mu > \mu_0 \). Then
\[ \left| \frac{\sqrt{n}(\mu_0 - \mu)}{\sigma} + z_{\alpha/2} \right| < \left| \frac{\sqrt{n}(\mu_0 - \mu)}{\sigma} - z_{\alpha/2} \right|. \]

Hence,
\[ \phi\left( \frac{\sqrt{n}(\mu_0 - \mu)}{\sigma} + z_{\alpha/2} \right) > \phi\left( \frac{\sqrt{n}(\mu_0 - \mu)}{\sigma} - z_{\alpha/2} \right). \]

Because \( \phi(t) \) is symmetric about 0, \( \phi(t) = \phi(|t|) \). This observation plus the last inequality shows that \( \gamma'(\mu) \) is increasing, (for \( \mu > \mu_0 \)). Likewise for \( \mu < \mu_0, \gamma' \) is decreasing.
4.6.5 (a). The critical region is

\[ t = \frac{\bar{x} - 10.1}{s/\sqrt{15}} \geq 1.753. \]

The observed value of \( t \),

\[ t = \frac{10.4 - 10.1}{0.4/\sqrt{15}} = 2.90, \]

is greater than 1.753 so we reject \( H_0 \).

(b). Since \( t_{0.008}(15) = 2.947 \) (from other tables), the approximate \( p \)-value of this test is 0.005.

4.6.7 Assume that \( X \) and \( Y \) are normally distributed. Then the \( t \)-statistic

\[ t = \frac{\bar{X} - \bar{Y}}{s_p \sqrt{1/n_1 + 1/n_2}} \]

has under \( H_0 \) a \( t \)-distribution with \( n_1 + n_2 - 2 \) degrees of freedom. A level \( \alpha \) test for the alternative \( H_A : \mu_1 < \mu_2 \) is

Reject \( H_0 \) in favor of \( H_A \), if \( t < -t_{\alpha, n_1+n_2-2} \).

For Part (b), based on the data we have,

\[ s_p^2 = \frac{(13 - 1)25.6^2 + (16 - 1)28.3^2}{27} \]
\[ s_p = \sqrt{s_p^2} = 27.133 \]
\[ t = \frac{72.9 - 81.7}{27.133 \sqrt{1/13 + 1/16}} = -0.8685. \]

Since \( t = -0.8685 \neq -t_{0.05,27} = -1.703 \), we fail to reject \( H_0 \) at level 0.05. The \( p \)-value is \( P[t(27) < -0.8685] = 0.1964. \)