Assignment 2 (Due Thursday, 01/24/08)

1. Two types of fertilizer mixture $A$ and $B$ are used to feed six bean plants with each randomly applied to three plants. The yields of the plants (in pounds) are given below. We want to test whether $A$ and $B$ have different effects on the yield of bean plants.

\begin{equation}
A \quad B \quad B \quad A \quad B \quad A
\end{equation}

\begin{equation}
3 \quad 5 \quad 4 \quad 3 \quad 7 \quad 6
\end{equation}

a) Construct the randomization distribution.
b) Calculate the $p$-value based on the randomization test.
c) Calculate the $p$-value based on two-sample $t$-test.
d) Compare the results from b) and c).

2. You are requested to design an experiment to compare the typing efficiency of three different types of keyboards denoted by $A$, $B$ and $C$. Two typists, denoted by $T_1$ and $T_2$, are employed and six standard manuscripts $m_1$, $m_2$, $m_3$, $m_4$, $m_5$ and $m_6$ are used.

a) Give your experimental plan.
b) Which principles you have used in designing the experiment? Comment on their advantages in this particular experiment.

3. a) We introduce a simple statistical model for the keyboard experiment discussed in class. Let $y$ be the amount of time used to type up a manuscript. Note that $y$ depends on keyboard, manuscript, whether the manuscript has already been typed, and experimental error. Let $\tau_A$ and $\tau_B$ denote the effects of keyboard $A$ and $B$ respectively, $\beta_i$ the effect of manuscript $i$ for $i = 1, 2, 3, 4, 5, 6$, and $\epsilon$ the experimental error. Let $\eta_l$ denote the learning effect. We
are interested in estimating the difference between $\tau_B$ and $\tau_A$. Suppose Design 2 from the lecture notes is used in the experiment, which is


The statistical model for the amount of time used in $1.A$, denoted by $y_{1A}$, is

$$y_{1A} = \mu + \tau_A + \beta_1 + \epsilon_{1A}, \quad (1)$$

and the model for the amount of time used in $1.B$ is

$$y_{1B} = \mu + \tau_B + \beta_1 + \eta_l + \epsilon_{1B}, \quad (2)$$

where $\mu$ is some constant.

a) Is $\eta_l$ positive or negative? Why it is included in (2) but not in (1)?

b) Write down the statistical models for the other runs.

c) How to estimate $\tau_B - \tau_A$, using $y_{1A}, y_{1B}, \ldots, y_{6A}, y_{6B}$.

d) Compared to Design 1 in the lecture, what is the gain (in terms of estimating $\tau_B - \tau_A$) from using randomization? Recall that the typing order of the manuscripts is randomized in Design 2.

e) Derive an experimental plan using balanced randomization as discussed in class. Is there any further gain of your plan over Design 2?