1. Let $X_1, X_2, X_3$ be independent exponential waiting times, each with an average of 30 minutes. Let $Y = X_1 + X_2 + X_3$.

a. What is the average (in minutes) of $Y$?

b. What is the standard deviation (in minutes) of $Y$?
2. A chef working in a kitchen believes that the waiting time until the next dessert order is exponential, with an average of 3 minutes. The times between dessert orders are assumed to be independent exponentials, also with 3 minutes on average. Let \( Y \) be the time until the next dessert order, and let \( Z \) be the subsequent time (afterwards) until the following dessert order.

[E.g., if it is 12 noon right now, and the next order arrives at 12:04 PM, and the order after that arrives at 12:11 PM, then \( Y = 4 \) and \( Z = 7 \).]

Let \( X = Y + Z \). Find the density of \( X \).
3. Suppose that the times until Hector, Ivan, and Jacob’s pizzas arrive are independent exponential random variables, each with average of 20 minutes. Let $X$ be the sum of the times that they spend waiting, i.e., Hector’s time plus Ivan’s time plus Jacob’s time. Find the variance of $X$. 
4. [Question about Exponential random variables.]

Let $X$ be exponential with expected value 3. Let $Y$ be another random variable that depends on $X$ as follows: if $X > 5$, then $Y = X - 5$; otherwise, $Y = 0$.

a. Find the expected value of $Y$.

b. Find the variance of $Y$. 
5. [Question about Exponential random variables.]

Suppose that Michelle, Nancy, and Olivia each are waiting for their husbands to appear. Their waiting times are assumed to be independent exponentials, and they each expect to wait 5 minutes. Let $X$ denote the time until the very first husband appears.

What is the expected value of $X$? [Hint: Since $X$ is the minimum of three independent exponential random variables, then $X$ is exponential.]
6. **Design your own problem and solution.** Create your own problem about a Gamma random variable. Design your problem in such a way that you would find it enjoyable and also interesting (i.e., not completely trivial) if you found this problem in a probability book. Please provide the answer for your problem too.