Problem Set 12

Instructions for this assignment: Solve any 5 of the problems #1 through #8 that you choose, and also solve problem #9. You can get some extra credit by solving the others too.

Show all steps for all of your solutions. Circling the correct solution is insufficient.

Name: ________________________________________________________________

1. Suppose that four urns each contain two balls. Urns 1 and 2 each have one black ball and one white ball. Urn 3 has two black balls. Urn 4 has two white balls. An urn is selected at random. Given that the first ball drawn from this urn is black, what is the probability that the other ball in this urn is also black?

A.) 1/8  B.) 1/6  C.) 1/4  D.) 1/3  E.) 1/2
2. A box has 10 balls, 6 of which are black and 4 of which are white. Three balls are removed from the box, their color unnoted. Find the probability that a fourth ball removed from the box is white.

A.) 1/10  B.) 1/4  C.) 1/3  D.) 2/5  E.) 2/3
3. In answering a question on a multiple-choice test, a student either knows the answer or guesses. Let $p = .70$ be the probability that the student knows the answer and $1 - p = .30$ the probability that the student guesses. Assume that a student who guesses at the answer will be correct with probability $1/5$, since 5 is the number of multiple-choice alternatives. What is the conditional probability that a student knew the answer to a question, given that he or she answered it correctly?

A.) .43  B.) .70  C.) .74  D.) .90  E.) .92
4. Urn I contains 2 black balls and 2 white balls; urn II contains 1 black ball and 2 white balls; urn III contains 3 black balls and 1 white ball. One urn is selected at random, and then one ball is drawn from it. Given that a black ball is drawn, what is the probability that urn I was selected?

A.) $\frac{2}{11}$  B.) $\frac{6}{19}$  C.) $\frac{2}{6}$  D.) $\frac{19}{36}$  E.) $\frac{6}{11}$
5. An urn contains 7 balls, of which 2 are white. Two players—$A, B$—successively draw from the urn, $A$ first, then $B$, then $A$, then $B$, then $A$, and so on. The winner is the first one to draw a white ball.

What is the probability $B$ wins if each ball is replaced after it is drawn?
What is the probability $B$ wins if the withdrawn balls are not replaced?

A.) with replacement, $P(B \text{ wins}) = \frac{2}{9}$; without replacement, $P(B \text{ wins}) = \frac{3}{7}$
B.) with replacement, $P(B \text{ wins}) = \frac{5}{12}$; without replacement, $P(B \text{ wins}) = \frac{3}{7}$
C.) with replacement, $P(B \text{ wins}) = \frac{5}{12}$; without replacement, $P(B \text{ wins}) = \frac{8}{21}$
D.) with replacement, $P(B \text{ wins}) = \frac{2}{9}$; without replacement, $P(B \text{ wins}) = \frac{5}{21}$
E.) with replacement, $P(B \text{ wins}) = \frac{5}{7}$; without replacement, $P(B \text{ wins}) = \frac{8}{21}$
6. Die $A$ has four red and two white faces, whereas die $B$ has two red and four white faces.

A fair coin is flipped once at the start of a game. If the coin lands heads, then die $A$ is used throughout the game. If the coin lands tails, then die $B$ is used throughout the game.

If the first $n$ throws of the die are red, what is the probability that die $A$ is being used?

A.) $\frac{2^n}{3^{n+1}}$  
B.) $(1/2)^n$  
C.) $(2/3)^n$  
D.) $\frac{2^n}{2^{n+1}}$  
E.) $\frac{3^n}{3^{n+1}}$
7. How many rolls of a die are needed so that the probability of “5” appearing at least once is at least 9/10?

A.) 2  B.) 6  C.) 9  D.) 13  E.) 16
8. Suppose that a die is rolled twice. Let $X$ denote the maximum value to appear in the two rolls. Find the expected value of $X$.

A.) 2.53  B.) 3.11  C.) 3.50  D.) 4.47  E.) 5.06
9. **Design your own problem and solution.** Create your own problem about a topic covered so far in STAT/MA 416. Design your problem in such a way that you would find it enjoyable and also interesting (i.e., not completely trivial) if you found this problem in a probability book. Please provide the answer for your problem too.