Chapter 38 Problems

1. While waiting for the bus on a snowy morning, the expected waiting time (including unusual delays for snow!), is 12 minutes. Use the Markov inequality to find an upper bound on the probability that the bus takes 15 minutes or more to arrive.

In the same scenario as above, assume that the standard deviation of the waiting time is 3. Use the Chebyshev inequality to find a bound on the probability that the bus takes between 6 to 18 minutes to arrive.
2. The average number of students in a class at Purdue is 31. Use the Markov inequality to find an upper bound on the probability that a class selected at random will have 40 or more students.

In the scenario above, now assume that the variance of the class size is 64. Use the Chebyshev inequality to find a bound on the probability that a class selected at random will have between 17 and 45 people.
3. With the same assumptions as in Question 2, three classes are independently selected at random. Let $A$ denote the event that all three of the classes have 40 or more students (i.e., 40 or more in each class). Find a bound on the probability of $A$. Hint: Separate $A$ into three independent events, find a bound on the probability of each, and then think about how to appropriately combine your bounds.

In the scenario above, let $B$ denote the event that all three classes selected at random will have between 20 and 42 people (i.e., 20 to 42 people in each class). Find a bound on the probability of $B$. Hint: Again, separate $B$ into three independent events, find a bound on each, and then recombine appropriately.
Review question: 4. Let $X_1, X_2, X_3$ be independent continuous random variables, each uniformly distributed in the interval $[0, 10]$. Let $Y$ denote the middle of the three values. Find the cumulative distribution function $F_Y(a) = P(Y \leq a)$ of the random variable $Y$.

Hint: There are six possible orderings of $X_1, X_2, X_3$. So let’s write $U_1$ for the smallest, $U_2$ for the middle, and $U_3$ for the largest. So $Y = U_2$ is the middle value. So to find $P(Y \leq a)$ or in other words $P(U_2 \leq a)$, we just have 6 ways to choose which $X$’s correspond to which $U$’s, and then we just need $U_2 \leq a$ and $U_1 \leq U_2$ and $U_3 \geq U_2$. So the desired integral is, for $0 \leq a \leq 10$,

$$F_Y(a) = P(Y \leq a) = P(U_2 \leq a) = 6 \int_0^a \int_0^{u_2} \int_{u_2}^{10} \left(\frac{1}{10}\right)^3 du_3 \, du_1 \, du_2.$$

I hope that I didn’t say too much!!
5. Create your own scenario with a random variable $X$ for which the distribution is not necessarily known. Create a Markov inequality bound on a probability that is interesting in your scenario.
6. Create your own scenario with a random variable $X$ for which the distribution is not necessarily known. Create a Chebyshev inequality bound on a probability that is interesting in your scenario.