Chapter 30 Problems

1. At a certain university, a large course has 1500 registered students; the course can be viewed online if a student does not feel like walking to the lecture hall. Each student decides each day (independently of the other students and independent of her/his own prior behavior) whether or not to attend the class in the lecture hall or just view the online version. The probability a student attends class in the lecture hall is only .30, so the registrar has quite a headache. There is no need to use a lecture hall with 1500 chairs, because the expected number of students to attend on a particular day is only \((.30)(1500) = 450\) (wow!).

If the registrar assigns a lecture hall with 500 seats for the class, what is the probability that the lecture hall overflows (i.e., strictly more than 500 attend) on a particular day?

To save money, the registrar wants to use an even smaller lecture hall next semester, but the professor insists on having enough seats so the probability is 95% (or more) of no overflow on a given day. How many seats does the professor need in her lecture hall?
2. Let’s revisit the scenario in Example 16.2 of Chapter 2, namely: Consider customers who arrive at a checkout counter with an average rate of 8 per hour. Assume that a Poisson number of customers arrive per hour. So, during an eight-hour shift, the average number of customers is 64.

Find the probability that strictly more than 70 customers arrive during the eight-hour shift.
3. Review question: [This one is not about approximation by normal random variables.] On Purdue license plates in the State of Indiana, there are always two letters (e.g., “PU”) followed by four digits, and thus 10,000 combinations of plates are available for each pair of letters. If the four digits are selected randomly, and all 10,000 possibilities are equally-likely, what is the probability that the four digits are distinct and in ascending order?
4. **Review question:** [This one is not about approximation by normal random variables.] Three children are skating around the perimeter of a circle in an ice rink of radius 50 feet. Assume that their locations around the perimeter of the circle are independent, continuous uniform random variables. Their mother comes to the door of the ice rink (located at a fixed position on perimeter of the circle). When she appears, what is the expected distance around the perimeter from her to the closest of the three children?

[Hint: The distance from the mother to a specific child is between 0 and 50π (i.e., half the circumference).]
5. Construct a scenario with one binomial random variable with parameters $n$ and $p$ such that $np(1 - p)$ is relatively large (e.g., $np(1 - p) \geq 10$). Use the Central Limit Theorem to calculate a probability that is interesting in your scenario.
6. Construct a scenario with one Poisson random variable with a relatively large parameter $\lambda$ (e.g., $\lambda \geq 10$). Use the Central Limit Theorem to calculate a probability that is interesting in your scenario.