Chapter 28 Problems

1. Consider a randomly-selected book with the assumptions from Example 28.3, namely, the book has expected thickness \( \mu = 1.125 \) (in inches) and standard deviation \( \sigma = .25 \). If the bookseller builds a shelf that is only 1.8 inches, what is the probability that her book will fit on it?
2. On a certain one-lane highway, the distance (in feet) between consecutive cars is assumed to be normally distributed with expected value \( \mu = 200 \) and standard deviation \( \sigma = 30 \). A traffic helicopter adds the distances between car 0, car 1, car 2, \ldots, car 26, and obtains the sum \( X_1 + \cdots + X_{26} \) (since 27 cars have 26 gaps in between). So \( X_j \sim \mathcal{N}(\mu = 200, \sigma = 30) \), and the \( X_j \)'s are assumed to be independent.

Find the probability that the distance between car 0 and car 26 is less than 1 mile, i.e., find the probability that

\[
X_1 + \cdots + X_{26} \leq 5280.
\]

[There are 5280 feet in 1 mile.]
3. As in Question 3 on Problem Set 27, assume that the annual precipitation in West Lafayette, Indiana, is normally distributed, with expected value $\mu = 36.3$ inches and variance $\sigma^2 = 8.41$. Also assume that the amount of precipitation is independent in distinct years.

Let $X_1, \ldots, X_{10}$ denote the precipitation in the 10 distinct years of a decade. Find the probability that the total rainfall during the decade exceeds 380 inches.
4. As in Question 4 on Problem Set 27, assume that the height (in inches) of an American female is normal with expected value $\mu_1 = 64$ and standard deviation $\sigma_1 = 12.8$. Also assume that the height of an American male is normal with expected value $\mu_2 = 70$ and standard deviation $\sigma_2 = 14.0$. Let $X$ denote the female’s height and $Y$ denote the male’s height.

[Notice: Since $X$ is normal, then $aX + b$ is normal for all $a, b$; in particular, with $a = -1$ and $b = 0$, we know $-X$ is normal too. Think: What is $E[-X]$? What is $\text{Var}(-X)$? Also, the sum of two normal random variables is normal, so $Y + (-X)$, a.k.a., $Y - X$, is normal too.]

Now, very carefully, find the expected value, variance, and standard deviation of $Y - X$.

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Find the probability that a randomly-selected male is taller than a randomly-selected female.
5. Review question: [This one is not about normal random variables.] David works at a customer call center. He talks to customers on the telephone. The length (in hours) of each conversation has density \( f_X(x) = 3e^{-3x} \) for \( x > 0 \) and \( f_X(x) = 0 \) otherwise. The lengths of calls are independent. As soon as one conversation is finished, he hangs up the phone, and immediately picks up the phone again to start another call (i.e., there are no gaps in between the calls). Thus, if he conducts \( n \) phone calls in a row, the total amount of time he spends on the telephone is \( X_1 + \cdots + X_n \), where the \( X_j \)'s are independent, and each \( X_j \) has the density above.

Find the expected value, the variance, and the standard deviation, of the amount of time that he spends on the telephone during \( n \) calls, namely, \( X_1 + \cdots + X_n \).

What is the probability that his completes the first two calls within the first 1 hour? In other words, what is \( P(X_1 + X_2 \leq 1) \)?
6. Describe a scenario that uses a collection of normal random variables of your own, called \( X_1, X_2, \ldots, X_n \) (choose whatever \( n \) you like; it should be somewhat large... say, use \( n \geq 10 \)).

For \( X_1 + \cdots + X_n \), list the expected value \( n\mu \), variance \( n\sigma^2 \), and standard deviation \( \sqrt{n\sigma^2} \).

Calculate a probability (of your choice) related to \( X_1 + \cdots + X_n \) (you will need to convert to a standard normal random variable to calculate the probability itself).