Chapter 26 Problems

1. A painter accidentally splashes three drops of paint onto a nearby picture. The locations of the drops are independent. Fortunately, if a drop lands on the glass, it can be easily removed... however, if one or more of the drops lands on the frame of the picture, the frame is ruined. The dimensions of the glass and frame are given in the figure below. What is the probability that the frame is ruined?
2. A photographer is taking pictures of runners in a race. The density of the distance between each runner and the photographer (at the time of the picture) is constant on the interval [1, 6], measured in feet. (Assume that the locations of the runners are independent.) The photographer’s “very good” pictures are taken when the object is less than 3 feet away. What is the probability that none of the photographers’ pictures are “very good”? 
3. For the scenario in Question 2, if $X, Y, Z$ are the distances of the photographer from the runners Alice, Bob, and Christine, respectively, then find the expected value of the smallest of these three distances. I.e., find $E[\min(X, Y, Z)]$. 

4. Kelly throws a dart at a circular dartboard of radius 2 feet. Let $X$ and $Y$ denote the location where the dart lands. Assume that $-2 \leq X \leq 2$ and $-2 \leq Y \leq 2$ and $X^2 + Y^2 \leq 4$, i.e., the dart lands on the dartboard. Moreover, assume that the dart’s location is uniform on the dartboard, i.e.,

$$f_{X,Y}(x, y) = 1/(4\pi) \quad \text{if } x, y \text{ are on the dartboard, i.e., } x^2 + y^2 \leq 4,$$

and $f_{X,Y} = 0$ otherwise.

Let $D = \sqrt{X^2 + Y^2}$ be the distance of the dart to the center of the dartboard. Find $E[D]$. 
5. Create an interesting scenario with a uniform density. Calculate an interesting probability that is related to your scenario.
6. Create your own scenario with a uniform density that involves two (or more) dimensions, i.e., an area or volume. Calculate an interesting probability that is related to your scenario.