Chapter 24 Problems

1. Consider the joint density of $X$ and $Y$ from Example 22.1 of Chapter 22, namely,

$$f_{X,Y}(x, y) = \frac{1}{8}(1 - x^2)(3 - y) \quad \text{for } -1 \leq x \leq 1 \text{ and } -1 \leq y \leq 1,$$

and $f_{X,Y}(x, y) = 0$ otherwise.

Find $E[X]$.

Find $E[Y]$. 
2. Consider the joint density of $X$ and $Y$ from Example 22.3 of Chapter 22, namely,

$$f_{X,Y}(x, y) = \frac{3}{2}xy \quad \text{for } 0 \leq x \text{ and } 0 \leq y \text{ and } x + y \leq 2,$$

and $f_{X,Y}(x, y) = 0$ otherwise.

Find $E[Y]$. 
3. Consider the joint density of $X$ and $Y$ from Example 23.1 of Chapter 23, namely:

A bird lands in a grassy region described as follows: $0 \leq x$, and $0 \leq y$, and $x + y \leq 10$. Let $X$ and $Y$ be the coordinates of the bird’s landing. Assume that $X$ and $Y$ have the joint density

$$f_{X,Y}(x, y) = \frac{1}{50}$$

for $0 \leq x$ and $0 \leq y$ and $x + y \leq 10$,

and $f_{X,Y}(x, y) = 0$ otherwise.

Find $E[Y]$.

[Hint: The answer is not “$E[Y] = 5$.”]
4. Consider two points that are independently placed on a line of length 10, at locations $X$ and $Y$. Thus the joint density of $X$ and $Y$ is

$$f_{X,Y}(x, y) = \frac{1}{100} \quad \text{for } 0 \leq x \leq 10 \text{ and } 0 \leq y \leq 10,$$

and $f_{X,Y}(x, y) = 0$ otherwise.

First, try to show that, if $Z$ denotes the distance between $X$ and $Y$ (so $0 \leq Z \leq 10$), then the cumulative distribution function $F_Z(z)$ of $Z$ is

$$F_Z(z) = \frac{1}{5}z - \frac{1}{100}z^2 \quad \text{for } 0 \leq z \leq 10$$

and $F_Z(z) = 0$ for $z \leq 0$ and $F_Z(z) = 1$ for $z \geq 10$. [Hint: Use the complement; try to find $P(Z > z)$, i.e., the probability that the distance between $X$ and $Y$ is more than $z$. A picture should help.]

Whether or not you can accomplish the part above, just find $f_Z(z)$, by differentiating $F_Z(z)$.

Use the density $f_Z(z)$ to find $E[Z]$, i.e., the expected distance between $X$ and $Y$. 


5. **Review question:**

Consider the baseball scenario discussed in Example 22.4 of Chapter 22, namely:

Each time a pitcher delivers a fast ball, the speed is distributed between 90 and 100 miles per hour, with density $1/10$. Assume that the speeds of pitches are independent.

On Monday, the pitcher throws fast balls repeatedly, until the very first fast ball that exceeds 98.7 miles per hour. How many fast balls does he expect to throw, to accomplish this task?

On Tuesday, the pitcher throws exactly 15 fast balls. How many of them does he expect to exceed 98.7 miles per hour?
6. Create your own continuous random variable $X$. Find the expected value of $X$. 