Chapter 20 Problems

1. Consider the function $f_X(x) = c(1 - x^2)$ for $-1 < x < 1$, and $f_X(x) = 0$ otherwise. Here $c$ is a constant. Find the value of $c$ so that $f_X(x)$ is a density function.
2. Consider the function from Example 20.3, namely, \( f_X(x) = 1/6 \) for \( 4 < x < 10 \), and \( f_X(x) = 0 \) otherwise.

What is the cumulative distribution function \( F_X(x) \), for \( x \leq 4 \)?

What is the cumulative distribution function \( F_X(x) \), for \( 4 < x < 10 \)?

What is the cumulative distribution function \( F_X(x) \), for \( x \geq 10 \)?

Draw a picture of the cumulative distribution function \( F_X(x) \).
3. Consider the function from Example 20.4, namely, $f_X(x) = 3e^{-3x}$ for $x > 0$, and $f_X(x) = 0$ otherwise.

What is the cumulative distribution function $F_X(x)$, for $x \leq 0$?

What is the cumulative distribution function $F_X(x)$, for $x > 0$?

Draw a picture of the cumulative distribution function $F_X(x)$. 
4. Again consider the function from Example 20.4, namely, \( f_X(x) = 3e^{-3x} \) for \( x > 0 \), and \( f_X(x) = 0 \) otherwise.

Find an expression for \( \int_{-\infty}^{\infty} xf_X(x) \, dx \).

[Hint: Use integration by parts. Also, keep in mind that the integral from \(-\infty\) to 0 is just 0, because \( f_X(x) = 0 \) for \( x \leq 0 \).]
5. Describe three continuous random variables $X$, $Y$, $Z$ (just to get you thinking about continuous random variables). They can be related or unrelated; it is completely up to you!

Here is the description for random variable $X$:

Here is the description for random variable $Y$:

Here is the description for random variable $Z$:
6. Give any nonnegative function that integrates to 1 over the whole real line. (You choose whatever function you like.) Call this function $f_X(x)$, since it is a density for some random variable $X$. Now give an expression for $F_X(x)$, which is the cumulative distribution function of $X$. 