Chapter 18 Answers

In each problem, all possibilities are equally likely. Each problem counts for 1.5 points altogether, i.e., each part of each problem is worth 0.5 points.

1. If Alice and Alan (a couple) and Barbara and Bob (another couple) sit in a row of chairs, what is the probability that each of the couples sit together?

There are 4! = 24 ways that the 4 people can be seated altogether. (This is true in all of the questions with 4 people, so we won’t repeat this fact.)

Grouping the couples together, there are 2! ways that the couples can be seated (i.e., either A’s collectively on the left or right). For each such way, there are 2 ways that the A’s can be arranged among themselves (Alice, Alan or Alan, Alice) and 2 ways that the B’s can be arranged among themselves (Barbara, Bob or Bob, Barbara). So the total probability that the couples are seated together is 2!(2)(2)/24 = 1/3.

If Alice and Alan (a couple) and Barbara and Bob (another couple) and Christine and Charlie (another couple) sit in a row of chairs, what is the probability that each of the 3 couples sit together?

There are 6! = 720 ways that the 6 people can be seated altogether. (This is true in all of the questions with 6 people, so we won’t repeat this fact.)

Grouping the couples together again, there are 3! ways that the couples can be seated (i.e., either A’s/B’s/C’s, or A’s/C’s/B’s, or B’s/A’s/C’s, etc... 3! ways total). For each such way, there are 2 ways that the A’s can be arranged among themselves, and 2 ways that the B’s can be arranged among themselves, and 2 ways that the C’s can be arranged among themselves. So the total probability that the couples are seated together is 3!(2)(2)(2)/720 = 1/15.

If \( n \) couples sit in a row of chairs, what is the probability that each of the \( n \) couples sits together?

There are \((2n)!\) ways that the \( 2n \) people can be seated altogether. (This is true in all of the questions with \( 2n \) people, so we won’t repeat this fact.)

Grouping the couples together again, there are \( n! \) ways that the couples can be seated. For each such way, there are 2 ways within each couple that the men and woman can be arranged in their two reserved seats. So the total probability that the couples are seated together is \( n!2^n/(2n)! \).
2. If Alice and Barbara (two girls) and Alan and Bob (two boys) sit in a row of chairs, what is the probability that the girls all sit together (the boys may or may not be in a group)?

There are 2! ways that the boys can be arranged. The girls, as a group, can be collectively put into any of the 3 gaps between the boys, including the spaces on the left- or right-hand ends. Once the girls are placed as a group, there are 2! arrangements among just the girls themselves. So the total probability that the girls sit together as a group is (2!)(3)(2!)/4! = 1/2.

If Alice, Barbara, and Christine (three girls) and Alan, Bob, and Charlie (three boys) sit in a row of chairs, what is the probability that the girls all sit together (the boys may or may not be in a group)?

There are 3! ways that the boys can be arranged. The girls, as a group, can be collectively put into any of the 4 gaps between the boys, including the spaces on the left- or right-hand ends. Once the girls are placed as a group, there are 3! arrangements among just the girls themselves. So the total probability that the girls sit together as a group is (3!)(4)(3!)/6! = 1/5.

If \( n \) girls and \( n \) boys sit in a row of chairs, what is the probability that the girls all sit together (the boys may or may not be in a group)?

There are \( n! \) ways that the boys can be arranged. The girls, as a group, can be collectively put into any of the \( n+1 \) gaps between the boys, including the spaces on the left- or right-hand ends. Once the girls are placed as a group, there are \( n! \) arrangements among just the girls themselves. So the total probability that the girls sit together as a group is \( n!(n+1)n!/(2n)! \).
3. If Alice and Barbara (two girls) and Alan and Bob (two boys) sit in a row of chairs, what is the probability that the girls all sit together and all the boys sit together?

Either the girls sit collectively on the right or on the left (i.e., 2 ways). Once the girls are placed as a group, there are 2! arrangements among just the girls themselves, and there are 2! arrangements among just the boys themselves. So the total probability that the girls all sit together and the boys all sit together $\frac{(2)(2!)(2!)}{4!} = \frac{1}{3}$.

If Alice, Barbara, and Christine (three girls) and Alan, Bob, and Charlie (three boys) sit in a row of chairs, what is the probability that the girls all sit together and all the boys sit together?

Either the girls sit collectively on the right or on the left (i.e., 2 ways). Once the girls are placed as a group, there are 3! arrangements among just the girls themselves, and there are 3! arrangements among just the boys themselves. So the total probability that the girls all sit together and the boys all sit together $\frac{(2)(3!)(3!)}{6!} = \frac{1}{10}$.

If $n$ girls and $n$ boys sit in a row of chairs, what is the probability that the girls all sit together and all the boys sit together?

Either the girls sit collectively on the right or on the left (i.e., 2 ways). Once the girls are placed as a group, there are $n!$ arrangements among just the girls themselves, and there are $n!$ arrangements among just the boys themselves. So the total probability that the girls all sit together and the boys all sit together $\frac{(2)(n!)(n!)}{(2n)!}$. 


4. If Alice and Barbara (two girls) and Alan and Bob (two boys) sit in a row of chairs, what is the probability that none of the girls are adjacent and none of the boys are adjacent?

Either the leftmost chair is for a girl or a boy (i.e., 2 ways). Afterwards, the gender of the people in every chair is determined. Once this is determined, there are 2! arrangements among just the girls themselves, and there are 2! arrangements among just the boys themselves. So the total probability that the girls all sit together and the boys all sit together \((2)(2!)(2!)/4! = 1/3\).

If Alice, Barbara, and Christine (three girls) and Alan, Bob, and Charlie (three boys) sit in a row of chairs, what is the probability that none of the girls are adjacent and none of the boys are adjacent?

Either the leftmost chair is for a girl or a boy (i.e., 2 ways). Afterwards, the gender of the people in every chair is determined. Once this is determined, there are 3! arrangements among just the girls themselves, and there are 3! arrangements among just the boys themselves. So the total probability that the girls all sit together and the boys all sit together \((2)(3!)(3!)/6! = 1/10\).

If \(n\) girls and \(n\) boys sit in a row of chairs, what is the probability that that none of the girls are adjacent and none of the boys are adjacent?

Either the leftmost chair is for a girl or a boy (i.e., 2 ways). Afterwards, the gender of the people in every chair is determined. Once this is determined, there are \(n!\) arrangements among just the girls themselves, and there are \(n!\) arrangements among just the boys themselves. So the total probability that the girls all sit together and the boys all sit together \((2)(n!)(n!)/(2n)!\).