Chapter 17 Answers

1. A recent survey states that 48% of mobile devices are iPhones. In order to learn more about how the iPhone works, a student starts asking his friends if they use an iPhone. What is the probability that one or more of the student’s closest three friends will use an iPhone?

Method #1. Let $X$ denote the number of all people that he needs to ask (starting with his 3 specific friends, but possibly going beyond them, to other people), until he finds the first iPhone user. Then $X$ is geometric with parameter $p = .48$. So

$$P(X > 3) = q^3 = (.52)^3 \approx 0.140608.$$ 

So the desired probability is

$$P(X \leq 3) = 1 - P(X > 3) = 1 - (.52)^3 = 0.859392.$$ 

Method #2. Let $X$ be the number of his specific 3 friends that have an iPhone, so $X$ is binomial with parameters $n = 3$ and $p = .48$. So

$$P(X = 0) = \binom{n}{0} p^0 q^3 = (.52)^3 \approx 0.140608.$$ 

Thus, the desired probability is

$$P(X > 0) = 1 - P(X = 0) = 1 - (.52)^3 = 0.859392.$$ 

2. A certain radio station plays songs from the 1970’s, 80’s, and 90’s. We know that 20% of the songs on the station are from the 70’s; 37% are from the 80’s; and 43% of the songs are from the 90’s. What is the expected value of the number of songs until the first song from the 1990’s is played?

The number of songs $X$ until the first song from the 1990’s is played is geometric with $p = .43$. So the expected number of songs until the first song from the 1990’s is played is $E[X] = 1/p = 1/.43 \approx 2.325581$.

Given that the upcoming five songs are a “rock block” from the 1970’s, what is the probability that strictly more than twelve songs will be needed to hear the first song from the 1990’s?

The conditional probability is

$$P(X > 12 \mid X > 5) = P(X > 7) = q^7 = .57^7 \approx 0.019549.$$ 

[Remember that we are able to use the rule about conditional probabilities of the form $P(X > k \mid X > j) = P(X > k - j)$ since $X$ is a geometric random variable.]
3. A young woman realizes that Homecoming is quickly approaching, and she needs to find a date. She estimates that 72% of the male students would be willing to accept her invitation. So she starts asking around; what is the expected number of men she will need to invite until she has a date for the Homecoming?

The number of men $X$ that she asks is geometric with $p = .72$. So the expected number of men she asks is $E[X] = 1/p = 1/.72 \approx 1.388889$.

Realizing that she finds a date very quickly with the method above, the young woman abandons her strategy (and she even abandons the man selected above).

The next day, she decides to assemble an all-new pool of twenty men who will accept her invitation (later, she will decide which of the twenty men she prefers!). What is the expected number of men that she needs to ask, until she has assembled a candidate pool of 20 men?

The number of men $Y$ that she asks is negative binomial with $k = 20$ and $p = .72$. So the expected number of men she asks, in order to assemble her pool of 20 men, is $E[X] = k/p = 20/.72 \approx 27.77778$.

4. A student particularly enjoys fresh fruit, but is willing to eat not-so-fresh fruit too. His strategy: Every day for breakfast, he eats fruit in the dining hall until he finds one that is fresh! Then he enjoys it, and finally heads to class. Only 60% of the pieces of fruit will be fresh enough for him to be happy! How many pieces of fruit does he expect to eat in a week? (The dining hall is open for breakfast 7 days a week.)

The number of pieces of fruit that he eats each day is geometric with parameter $p = .60$. So the number of pieces of fruit $X$ that he eats in a week is negative binomial, with parameters with $k = 7$ and $p = .60$. The expected number of pieces of fruit that he eats is

$$E[X] = q/p^2 = 7/.60 = 35/3 \approx 11.6666667.$$  

What is the variance of the number of pieces of fruit that he expects to eat in a week?

The variance of the number of pieces of fruit that he eats is

$$\text{Var}(X) = kq/p^2 = (7)(.40)/(.60)^2 = 70/9 \approx 7.7777778.$$