Chapter 16 Problems

1. Consider a Poisson random variable $X$ with parameter $\lambda = 5$. What is the exact probability that $X > 2$?

Now use your calculator to give this answer numerically (i.e., type the exact answer into your calculator, and give the result).
2. Suppose that the number of Roseate Spoonbills (a very rare bird in Indiana) that fly overhead in 1 hour has a Poisson distribution with mean 2. Also suppose that the number of Roseate Spoonbills is independent from hour to hour (e.g., the number of birds between noon and 1 PM does not affect the number of birds between 1 PM and 2 PM, etc.).

A bird watcher sits and looks for the birds for 3 hours. How many of these birds does she expect to see?

What is the probability that she sees exactly 5 of these birds during a 3 hour time period?
3. In a person’s lifetime, he plays a lottery game 10,000 times, but his chances of winning each time are only 1 in 5,000. Approximate the probability that he wins at least one time during his lifetime.
4. There is a certain disease in America which is very rare. Each person has a probability of 1 in 100,000,000 of having the disease. Assume that there are 310,000,000 people in America.

Approximate the probability that nobody in America actually has the disease.

Approximate the probability that at most 3 people (i.e., 3 or less) in America have the disease.
5. Create your own scenario, with a binomial random variable $X$. Make sure that your story has a very large $n$, a very small $p$, and a relatively reasonably-sized $np$. Give an exact expression for $P(X = 4)$. It will probably not be possible for you to enter the expression on your calculator, and that’s OK. Just write the exact expression. (Then take a look at question 6.)
6. For the scenario in Problem 5, use a Poisson approximation to approximate the probability that $X = 4$ in your example. Compute the approximate value, using your calculator.