Chapter 12 Problems

1. Jack and Jill are independently struggling to pass their last (one) class required for graduation. Jack needs to pass Calculus III, but he only has probability .30 of passing. Jill needs to pass Advanced Pharmaceuticals, but she only has probability .46 of passing. They work independently.

As in Problem Set 11, let $X$ be 0, 1, or 2, if (respectively) neither, one, or both of them graduate. Let $Y$ indicate if Jack graduates, so $Y = 1$ if Jack graduates, and $Y = 0$ otherwise. Let $Z$ indicate if Jill graduates, so $Z = 1$ if Jill graduates, and $Z = 0$ otherwise. Notice that $X = Y + Z$ always. Now find $E[X]$ using $E[Y]$ and $E[Z]$.

(The solution should be quick and easy, and you know from Problem Set 11 that $E[X]$ will be .76, of course!)
2. Four students order noodles at a certain local restaurant. Their orders are placed independently. Each student is known to prefer Japanese pan noodles 40% of the time (it is a very popular and tasty dish!). How many of them do we expect to order Japanese noodles?

Let $A_1, A_2, A_3, A_4$ be the events that (respectively) the first, second, third, or fourth person orders Japanese noodles. Let $X_1, X_2, X_3, X_4$ be indicator random variables for (respectively) $A_1, A_2, A_3, A_4$. Please justify your answer using the values of the $E[X_j]$'s.
3. Three hundred little plastic yellow ducks are dumped in a pond; one of them contains a prize tied around its foot. Leonardo examines each duck until he discovers the prize. He discards each duck without a prize after he checks it, so that he never needs to check a duck more than one time!

How many ducks does he expect to check until he discovers the prize? Please carefully justify your answer.
4. The Super Breakfast Challenge (SBC) consists of bacon, eggs, oatmeal, orange juice, milk, and several other foods, and it costs $12.99 per person to order at a local restaurant. It is known to be very difficult to consume the entire SBC. Only 10% of people are able to eat all of the SBC. The other 90% of people will be unable to eat the whole SBC (it is too much food!).

A probability student hears about the SBC and goes to the local restaurant. He observes the number of customers, \( X \), that attempt to eat the SBC, until the first success. So if there are 4 failures and then 1 success (i.e., the outcome is \((F, F, F, T)\)), then \( X = 5 \).

Find the expected value of \( X \), i.e., the number of customers expected to try the SBC until the first success. Justify your answer completely, using one of the methods from Chapter 12.
5. Create your own scenario with a discrete random variable $X$ that has a finite number of possible values. Compute the expected value of $X$ using one of the methods from Chapter 12.
6. Create another scenario of your own, with a discrete random variable $X$ that has infinitely many possible values (similar to the SBC example, or to Example 12.6, 12.7, or 12.8, for instance). Compute $E[X]$ using one of the methods from Chapter 12.