Chapter 10 Problems

1. Ten students sign up for a job opening, but only 1 of the students will be selected. The employer chooses randomly; all ten outcomes are equally likely. If person 3, 5, 7, or 9 gets the job, let $X = 1$; otherwise, $X = 0$. If person 1, 2, 3, 4, or 5 gets the job, let $Y = 1$; otherwise, $Y = 0$. Are $X$ and $Y$ independent random variables? Justify your answer.
2. Each day, Maude has a 1% chance of losing her cell phone (her behavior on different days is independent). Each day, Maude has a 3% chance of forgetting to eat breakfast (again, her behavior on different days is independent). Her breakfast and cell phone habits are independent.

Let $X$ be the number of days until she first loses her cell phone. Let $Y$ be the number of days until she first forgets to eat breakfast. (Here, $X$ and $Y$ are independent.) Find the joint mass of $X$ and $Y$. 
3. A student flips a fair coin until heads appears. Let $X$ be the numbers of flips until (and including) this first head. Afterwards, he begins flipping again until he gets another head. Let $Y$ be the number of flips, after the first head, up to (and including) the second head. E.g., if the sequence of flips is TTTTTTHTTH then $X = 7$ and $Y = 3$.

Are $X$ and $Y$ independent? Justify your answer.
4. Same scenario as problem 3. Let $Z$ be the total number of flips until (and including) the second head. So $Z = X + Y$; e.g., in the example given, $Z = 10$. Are $X$ and $Z$ independent? Justify your answer.
5. Create your own scenario with two independent random variables $X$ and $Y$. Show that they are independent.
6. Now create a scenario with two dependent random variables, $X$ and $Y$. Show that they are dependent, i.e., why they fail to be independent.