Chapter 7 Answers

1. Jack and Jill are independently struggling to pass their last (one) class required for graduation. Jack needs to pass Calculus III, but he only has probability .30 of passing. Jill needs to pass Advanced Pharmaceuticals, but she only has probability .46 of passing. They work independently. What is the probability that at least one of them gets a diploma?

Let $A, B$ denote the events that Jack and Jill (respectively) graduate. Then we want $P(A \cup B)$. By inclusion-exclusion, this is equal to

$$P(A \cup B) = P(A) + P(B) - P(A \cap B).$$

We are given $P(A) = .30$ and $P(B) = .46$. Since $A$ and $B$ are independent, then $P(A \cap B) = P(A)P(B) = (.30)(.46) = .138$. So we conclude

$$P(A \cup B) = .30 + .46 - .138 = .622.$$

2. People are selected randomly and independently for a drug test. Each person passes the test 98% of the time. What is the probability that the first 19 people pass, and the 20th person fails?

Write $A_j$ for the event that the $j$th trial is a success, and $B$ for the event that the 20th trial is a failure. The desired probability is $P(A_1 \cap A_2 \cap \cdots \cap A_{19} \cap B)$. Since the trials are independent, we compute

$$P(A_1 \cap A_2 \cap \cdots \cap A_{19} \cap B) = P(A_1)P(A_2)\cdots P(A_{19})P(B) = (.98)^{19}(.02) = (.98)^{19}(.02)$$
3. Ten students order noodles at a certain local restaurant. Their orders are placed independently. Each student is known to prefer Japanese pan noodles 40% of the time (it is a very popular and tasty dish!). Since the probabilities in parts a, b, c are very small, please write an exact expression for your answer. You do not need to simplify.

(a.) What is the probability that all ten of the students order Japanese pan noodles?

Let \( A_j \) be the event that the \( j \)th student orders Japanese pan noodles. The desired probability is \( P(A_1 \cap A_2 \cap \cdots \cap A_{10}) \). Since the students’ orders are independent, we compute

\[
P(A_1 \cap A_2 \cap \cdots \cap A_{10}) = P(A_1)P(A_2) \cdots P(A_{10}) = \left( \frac{2}{5} \right)^{10} = 0.0001048576
\]

(b.) What is the probability that none of the students order Japanese pan noodles?

The desired probability is \( P(A_1^c \cap A_2^c \cap \cdots \cap A_{10}^c) \). Since the students’ orders are independent, then the events \( A_1^c, A_2^c, \ldots, A_{10}^c \) are independent too, so we can compute

\[
P(A_1^c \cap A_2^c \cap \cdots \cap A_{10}^c) = P(A_1^c)P(A_2^c) \cdots P(A_{10}^c) = \left( \frac{3}{5} \right)^{10} = 0.0060466176
\]

(c.) What is the probability that at least one of the students orders Japanese pan noodles?

The event that at least one student gets Japanese pan noodles is just the complement of the event described in part b (where none of them get Japanese pan noodles). So the desired probability is \( 1 - (3/5)^{10} = 0.9939533824 \).

4. Is it possible to have two independent events \( A \) and \( B \) with the property that

\[
P(A) + P(B) > 1
\]

If your answer is “no”, give a brief justification of why this is impossible.

If your answer is “yes”, please give a brief example, in which you list the numbers \( P(A) \) and \( P(B) \) and also \( P(A \cup B) \) and \( P(A \cap B) \).

Yes, it is definitely possible. Consider, for instance, a situation similar to Problem #1, but now suppose that Jack has a 90% chance of graduating and also Jill has a 90% chance of graduating. Then \( P(A) = .90 \) and \( P(B) = .90 \). This makes \( P(A) + P(B) = 1.80 > 1 \).

Of course, \( A \) and \( B \) are not disjoint, so we did not break any of the rules of probability—there is a lot of overlap between \( A \) and \( B \) in our little example. Indeed, \( P(A \cap B) = P(A)P(B) = (.90)(.90) = .81 \); we multiply the probabilities since \( A \) and \( B \) are independent in this scenario. So both \( A \) and \( B \) happen simultaneously with probability .81. Also \( P(A \cup B) = P(A) + P(B) - P(A \cap B) = .90 + .90 - .81 = .99 \). So at least one of the students will graduate (in our new example) 99% of the time.