Small-Sample Tests – Proportions – Corrections are in Red

What I have in my notes is not correct for the upper-tailed and two-tailed. What I have in my notes is correct for the lower-tailed; however, I am not positive that I stated it correctly. I am just going to discuss the rejection region using a significance level of 0.1.

The following table is from Table A.1 in the book with the correct column outlined in purple. For our example, we have \( n = 10 \) and \( p_0 = 0.30 \)

<table>
<thead>
<tr>
<th>( p )</th>
<th>( 0.01 )</th>
<th>( 0.05 )</th>
<th>( 0.10 )</th>
<th>( 0.20 )</th>
<th>( 0.25 )</th>
<th>( 0.30 )</th>
<th>( 0.40 )</th>
<th>( 0.50 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.01</td>
<td>0.904</td>
<td>0.959</td>
<td>0.349</td>
<td>0.107</td>
<td>0.056</td>
<td>0.028</td>
<td>0.006</td>
<td>0.001</td>
</tr>
<tr>
<td>0.05</td>
<td>0.996</td>
<td>0.914</td>
<td>0.736</td>
<td>0.376</td>
<td>0.244</td>
<td>0.149</td>
<td>0.046</td>
<td>0.011</td>
</tr>
<tr>
<td>0.10</td>
<td>1.000</td>
<td>0.988</td>
<td>0.930</td>
<td>0.678</td>
<td>0.526</td>
<td>0.383</td>
<td>0.167</td>
<td>0.055</td>
</tr>
<tr>
<td>0.20</td>
<td>1.000</td>
<td>0.999</td>
<td>0.987</td>
<td>0.879</td>
<td>0.776</td>
<td>0.650</td>
<td>0.382</td>
<td>0.172</td>
</tr>
<tr>
<td>0.25</td>
<td>1.000</td>
<td>1.000</td>
<td>0.998</td>
<td>0.967</td>
<td>0.922</td>
<td>0.850</td>
<td>0.633</td>
<td>0.377</td>
</tr>
<tr>
<td>0.30</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td>0.994</td>
<td>0.980</td>
<td>0.953</td>
<td>0.834</td>
<td>0.623</td>
</tr>
<tr>
<td>0.40</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td>0.999</td>
<td>0.996</td>
<td>0.989</td>
<td>0.945</td>
<td>0.828</td>
</tr>
<tr>
<td>0.50</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td>0.999</td>
</tr>
</tbody>
</table>

In all of the cases, we want to find the largest rejection region possible.

**Lower-Tailed**

This is the easiest case to describe so I will do this one first.

For the lower-tailed case, \( z \leq c \) and we will reject if \( x \in \{0, ..., c\} \)

If we want the largest rejection region possible, we want \( c \) to be as large as possible so that \( B(c; n, p_0) \leq \alpha \). Therefore, if we increase to \( c + 1 \), we will have \( B(c + 1; n, p_0) > \alpha \). Therefore, we want to consider the case:

\[
B(c; n, p_0) \leq \alpha < B(c+1; n, p_0).
\]

Example: If \( n = 10 \), \( p_0 = 0.3 \), the \( c \) that fits inequality (1) is \( c = 0 \).

\[
B(0; 10, 0.3) = 0.028 \text{ and } B(1; 10, 0.3) = 0.149
\]

Therefore, the rejection region is \( x \in \{0\} \)
**Upper-Tailed**

For the upper-tailed case, \( z \geq c \) and we will reject if \( x \in \{c, \ldots, n\} \)

If we want the largest rejection region possible, we want \( c \) to be as small as possible so that
\[
P(X \geq c \text{ with } X \sim \text{Bin}(n, p_0)) \leq \alpha \]
since we decrease to \( c - 1 \), we will have
\[
P(X \geq c - 1 \text{ with } X \sim \text{Bin}(n, p_0)) > \alpha.
\]

\[
P(X \geq c \text{ with } X \sim \text{Bin}(n, p_0)) = 1 - P(X < c \text{ with } X \sim \text{Bin}(n, p_0)) = 1 - P(X \leq c - 1 \text{ with } X \sim \text{Bin}(n, p_0)) = 1 - B(c - 1; n, p_0)
\]

Similarly \( P(X \geq c - 1 \text{ with } X \sim \text{Bin}(n, p_0)) = 1 - B(c - 2; n, p_0) \)

So we want to consider \( 1 - B(c - 1; n, p_0) \leq \alpha < 1 - B(c - 2; n, p_0) \).

To make this easier to work with the following manipulations can be performed.

\[
1 - B(c - 1; n, p_0) \leq \alpha < 1 - B(c - 2; n, p_0) \\
1 - B(c - 1; n, p_0) - 1 \leq \alpha - 1 < 1 - B(c - 2; n, p_0) - 1 \\
- B(c - 1; n, p_0) \leq \alpha - 1 < - B(c - 2; n, p_0) \\
B(c - 2; n, p_0) < 1 - \alpha \leq B(c - 1; n, p_0)
\]

(2)

Example: If \( n = 10, p_0 = 0.3 \), the \( c \) that fits inequality (2) is \( c = 6 \).

\( B(6 - 2; 10, 0.3) = 0.850 \) and \( B(6 - 1; 10, 0.3) = 0.953 \) for a sum of \( 0.028 + 0.953 = 0.075 \).

If you would increase \( c_1 \) or decrease \( c_2 \), the sum would be greater than 0.10.

Therefore, the rejection region is \( x \in \{6, 7, 8, 9, 10\} \)

**Two-Tailed**

For the two-tailed case, \( z \leq c_1 \) or \( z \geq c_2 \) reject if \( x \in \{0, \ldots, c_1\} \cup \{c_2, \ldots, n\} \)

If we want the largest rejection region possible, we want \( c_1 \) to be as large as possible and \( c_2 \) to be as small as possible so that
\[
B(c_1; n, p_0) + P(X \geq c_2 \text{ with } X \sim \text{Bin}(n, p_0)) \leq \alpha \text{ or } B(c_1; n, p_0) + B(c_2 - 1; n, p_0) \leq \alpha
\]

(3)

Example: If \( n = 10, p_0 = 0.3 \), the \( c \)’s that fit inequality (3) are \( c_1 = 0 \) and \( c_2 = 6 \).

\( B(0; 10, 0.3) = 0.028 \) and \( B(6 - 1; 10, 0.3) = 0.953 \) for a sum of \( 0.028 + 1 - 0.953 = 0.075 \).

If you would increase \( c_1 \) or decrease \( c_2 \), the sum would be greater than 0.10.

Therefore, the rejection region is \( x \in \{0, 6, 7, 8, 9, 10\} \)