Examples for Counting Techniques

1) Roll two 8-sided dice. What is the probability that the sum of the two numbers is 5? (0.0625)

\[ N(A) = 4 \]
\[ \text{Possibilities (1,4), (2,3), (3,2), (4,1)} \]
\[ N = 8^2 \text{ (product rule)} \]
\[ P(A) = \frac{N(A)}{N} = \frac{4}{64} = 0.0625 \]

2) Draw two cards from a suit of 13 cards (say diamonds), what is the probability that the sum of the two cards is even? (A = 1, J = 11, Q = 12, K = 13)? (0.462)

\[ N(A) \text{: unordered without replacement, the sum is even if both are odd or both are even} \]
\[ N(A) = \text{number for both even } + \text{ number for both odd} = \binom{7}{2} + \binom{6}{2} = \frac{7!}{5!2!} + \frac{6!}{4!2!} = 21 + 15 = 36 \]
\[ N \text{: unordered without replacement, all possibilities of drawing 2 cards from 13} \]
\[ N = \binom{13}{2} = \frac{13!}{11!2!} = 78 \]
\[ P(A) = \frac{N(A)}{N} = \frac{36}{78} = 0.462 \]

3. The IRS decides that it will audit the returns of 3 people from a group of 18. If 8 of the people are women, what is the probability that all 3 of people audited are women?

\[ N(A) = \binom{8}{3} = \frac{8!}{(8-3)!3!} = 56 \]
\[ N = \binom{18}{3} = \frac{18!}{(18-3)!3!} = 816 \]
\[ P(A) = \frac{N(A)}{N} = \frac{56}{816} = 0.0686 \]

4. Arizona plates consist of three digits followed by three letters. What is the probability that a particular license plate doesn't have any repeating digits or letters?

\[ N(A) = P_{3,10} P_{3,26} = (10)(9)(8)(26)(25)(24) = 11,232,000 \]
\[ N = 10^326^3 = 17,576,000 \]
\[ P(A) = \frac{11,232,000}{17,576,000} = 0.639 \]