The Equity Premium Puzzle
Models and Forecasts

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Abstract

The equity premium can be loosely defined as the difference between the return on stocks and return on bonds. Unfortunately, there is no universally accepted definition of the equity premium and therefore it can be computed using different parameters and different historical time periods. These different methods of computing equity premium can lead to historical equity premium quotations ranging from 4.3% per year up to about 9.4% per year. In the past, the observed equity premium has consistently been much greater than the expected value. This is called the equity premium puzzle, for example an investment of $1 in stocks in 1925 would be worth $5,116 by 1998, whereas an investment in treasury bills would only be worth $15[1]. As the value of the equity premium is the prime input both in the CAPM and in asset allocation an accurate value is essential. In this project we have describe the various models that have been proposed to explain the behavior of equity premium. We also study the forecasts on the behavior of the equity premium in the near future.

[1] Data obtained from Ibbotson and Associates

I. Introduction: Equity Premium Puzzle

This project intends to study the models and forecasts to explain the equity premium puzzle. Loosely speaking, the equity premium is the difference between the return on risky stocks and return on safe bonds. It is the prime input both in the CAPM (the model used to compute an appropriate hurdle rate), and in asset allocation decisions (the choice of whether an investor should hold stocks or bonds).

Unfortunately, there is no universally accepted definition of the equity premium. In particular, one can compute equity premia using different stock market indices, different bonds, different methods of compounding or cumulating returns over time, and different historical time periods. Such computational differences can lead to valid historical equity premium quotations ranging from 4.3% per year up to about 9.4% per year, depending also on the time period quoted. It is important for a user of equity premium estimates to be clear about which definition is used and why it is the appropriate definition for the particular purpose it is used for.

There are three inter-related questions of primary interest:

1. **Why has the historical equity premium been so high?**
2. **What is a good prediction for the equity premium for the long run?**
3. **Can one use other variables, such as dividend yields, to come up with a good forecast of the equity premium, at least over a 1-5 year period?**

As first pointed out by Mehra and Prescott [1985], a return differential of, say, 8% per year is enormous especially over a long enough horizon. Ibbotson and Associates, the provider of historical equity premium data, show that an investment of $1 in 1925 would be worth $5,116 by 1998, whereas an investment in treasury bills would be worth $15.

The profession falls into three loosely defined camps with respect to why equity premia have been so high and whether they will continue to be high. The first camp argues that these high equity premia were necessary to induce investors to participate in the stock market (e.g., Benartzi and Thaler [1995], Campbell and Cochrane [1999]). This argument implies an equity premium (possibly) as high in the future as in the past.

The second camp argues that the expected returns necessary to entice investors have fallen in the last few years. This means, stocks can trade for much higher prices today, which itself is both responsible for the recent stock market appreciation, and for the expectation of much lower returns in the future.

The third camp argues that the stock market has gone crazy and we are all in a bubble right now. Again, this implies much lower future returns in the future. In contrast to the second camp, this camp would be less surprised by a crash or crash-like rapid drop in the stock-market.

One can also bring additional evidence to bear on the second question without taking a stance on the first question: There are a number of publicly accessible forecasts from investors, academics, corporations. Typically, investors seem to expect higher equity premia, academics seem to expect equal (or just slightly lower) equity premia, and corporations and consulting firms seem to expect lower equity premia than those realized in the past. In sum, these forecasts run counter to the view that equity premia have recently been so high, because everyone expects them to be lower in the near future. Finally, a new set of working papers are attempting to attribute equity premia based on long-term forecasts of real growth.

The third question is equally tricky as the first two questions. There is universal consensus that the equity premium cannot be easily predicted over shorter horizons, such as one month; and that even if the equity premium can be predicted over a longer horizon (e.g., over a 30-year forecast), we do not have enough historical data to run regressions to validate 30-year forecasting claims.

The variable most often mentioned as a possible predictor of equity premia is the prior-year dividend-yield. This predictability was first explored in a rigorous manner by the seminal papers of Campbell and Shiller [1988], Fama and French [1988], and Blanchard [1993], which came to the conclusion that dividend-yields do seem to have predictive ability. In a simple regression predicting equity premia one-year ahead with dividend-yields, the dividend yield shows great statistical significance. However, a long line of subsequent research has pointed to various statistical issues in this early work. Furthermore, most such models predict one-year-ahead forecasts of negative equity premia as of the year 2000-not a good prediction. Yet, even if there is disagreement of whether documented forecasting ability of dividend yields was real, there is little disagreement that this predictive ability has disappeared in the 1990s (Goyal-Welch [2000]).

In short, the equity premium is not only the single most important number of finance, but estimating it is also our most perplexing problem. If the profession fails to make progress in understanding the process driving the equity premium, progress on many of the most important problems in finance like proper asset allocation and hurdle rates are likely to be pseudo victories only.[2]

II. Models to Solve the Puzzle

Several models have been proposed by various researchers to match observational values with the theoretical ones. Among the most prominent are the following.

1. Standard Expected Utility Model (Mehra and Prescott [1985])

In their seminal paper, Mehra and Prescott (1985) employ a variation of Lucas (1978) pure exchange economy to examine an empirical problem facing the "representative" agent.[3] Each consumer in such economy maximizes his utility function

$$U_t = E_0[\ \textstyle\sum_{t=0} \beta^t U(c_t))\ ]\ ,\ 0<\beta<1 \qquad (1)$$

subject to a budget constraint.

[2] Source: Research Roundtable-The Equity Premium- Ivo Welch

[3] In Lucas' model, there is a perished single good and an asset in a one-good pure exchange economy with identical consumers. A single good is costly produced in a number of different productive units, while an asset is a claim to the output of one of these units. Equilibrium asset prices fluctuate stochastically through time as well as productivity in each unit does. This economy described is so called "representative" agent models.

They further restrict the utility function to be of constant relative risk aversion class taking in the form of

$$U(c, \alpha) = (c^{1-\alpha}) / (1-\alpha), 0 < \alpha \qquad (2)$$

where c represents the consumption of the consumer. β is a discount factor applied to the utility derived from future consumption. Hence, equation (1) implies that, at time t, each consumer, with identical preference, seeks to maximize the expectation discounted flow of utility over time. Mathematically, α is a measure of curvature of utility function; economically, a represents the consumer's attitude towards consumption profile. A larger a implies that individuals want consumption in different "state" to be more highly similar and also want consumption in different "dates" to be highly similar as well. The former implies that consumers dislike risk, the latter indicates that they dislike growth in their consumption profile.

According to their model, the maximum equity premium is 1.4%, which is much smaller than the historically observed 6.18% observed. Hence we have a puzzle that risk consideration alone cannot account for. This marked the start of the equity premium puzzle. We will discuss the some major modification models which try to resolve the puzzle in the following section.

2. Generalized Expected Utility Model (Epstein and Zin[1989, 91], Weil's [1990])

By modifying the preference of the individual consumer in Mehra and Prescott's model, Epstein and Zin describe a "generalization" of the standard expected utility. The utility function takes the form of

$$U_t = [ct^{1-\rho} + \beta\, (E_t\, U_{t+1}{}^{1-\alpha})^{(1-\rho)/(1-\alpha)}]^{1/(1-\rho)} \qquad (3)$$

Notably, (1) is a special case of (3) by setting up $\alpha = \rho$. This is why (3) is a "generalization" of the standard expected utility described in (1). Economically, (3) differ from (1) in that, as pointed out by Epstein and Zin, (3) may help explain various aspects of asset pricing behavior that appear anomalous in the context of (1) since it disentangles risk aversion and intertemporal substitution. Therefore, the individual attitude toward risk and growth are no longer governed by the same parameters α and $1/\alpha$ but two different parameters, α and $1/\rho$. Although the Epstein and Zin claim that their model explains the equity premium puzzle, some researchers believe that this model has potential to explain the risk-free rate puzzle.[4] Compared to the historical observed average equity premium of 6.2% and the largest equity premium of 1.4% predicted by Mehra and Prescott (1985), Epstein and Zin's results show an average equity premium of around 2%.

3. Habit Formation (Constantinides [1990])

This formulation assumes that utility is affected not only by current consumption but also by past consumption. In this model, the utility is a decreasing function of past consumption and marginal utility is an increasing function of past consumption.[5]

$$\mathbf{U(c)} = \boldsymbol{E_t \left[\sum_{s=0} \beta^s (c_{t+s} - \lambda\, c_{t+s-1})^{1-\alpha} / (1-\alpha) \right]} \qquad \textit{(4)}$$

where $\boldsymbol{\lambda}$ ***(>0)*** *captures the effect of past consumption*.

The property of this intertemporal preference is termed "habit formation". This model also explains the risk-free rate puzzle, but fails to solve the equity premium puzzle without invoking extreme aversion to consumption risk.

4. Idiosyncratic Risk to Aggregate Consumption (Mankiw (1986)

The models in the previous all assume that consumers are homogeneous. Instead, Mankiw (1986) assumes that there are an infinite number of individuals that are identical *ex ante*, however, that there consumption is not the same *ex post*. Another critical assumption he makes is that aggregate shocks to consumption are not equally dispersed across all consumers and these shocks only affect some consumers *ex post*. In incomplete markets using aggregate consumption

[4] See Kocherlakota (1996) and Mehra and Prescott (2003) for proof. "Risk-free rate puzzle" was proposed by Weil (1989).

[5] A good example of explaining this utility function was given in Kocherlakota (1996), where a consumer buying a BMW rather than a Yugo today will definitely feel better off today; however, having the BMW "spoils" the buyer and results in reducing his utility from all future car purchases.

data to estimate the equity premium becomes inappropriate because individual consumption is more variable than per capita consumption.

Under this framework, Mankiw concludes that unless aggregate shocks to income affect all investors equally *ex post*, relative asset returns in general depend on the distribution of aggregate shocks among the population. Hence, it is impossible to infer investors' risk aversion from the aggregate data alone. This may possibly explain the puzzle Mehra and Prescott have when the equity premium is estimated when per capita consumption data alone was used *ex post*.[6]

5. **Uninsurable Income Risk (Heaton and Lucas [1996, 1997])**

Another similar model submitted by Heaton and Lucas [1996, 1997] recognizes that the consumers face the risk of a job loss or other major personal disasters that cannot be hedged or insured against. Equities and related investments whose payoffs are contingent on the business cycle drop in value when the probability of job loss increases- as it does in recession, for instance. Thus in economic depression, consumers need an extra incentive to hold equities and similar investment instruments. This extra incentive can be quantified as the equity premium.

6. **Borrowing Constraints (Constantinides, Donaldson and Mehra [2002])**

The main insight in this paper is that the attractiveness of an asset changes over the lifetime of an individual as the correlation of equity with consumption changes. A young person has financial uncertainties like uncertain wage etc. and hence equity will be a hedge for this person. The same asset has a very different meaning for a middle-aged, financially secure person. They have to plan for the future when their retirement wage will be deterministic and the fluctuations in their consumption occur in correlation to the equity income. Hence, the characteristics of equity of an asset depend on the equity holder, and hence should influence the asset pricing. According to the authors the young people, who want to hold the asset, are shut out of the market due to borrowing constraints, the equity is therefore exclusive priced by the middle-aged people who

[6] This paragraph is mainly from Wang (2002).

would benefit if it is high. Hence, the equity premium is high and it continues to do so because of the middle-aged people who will not price it at a cheaper price that what they expect. Although this method explains the equity, it cannot explain the equity premium.

7. Taxes (McGrattan and Prescott [2000])

The people proposing this model argue that the equity premium can be explained based on the changes in tax rate. One implication of the model is that it predicts that as dividend yield falls and tax rates level out, equity returns will decline. As the dividend yield has changed little in the postwar period, it is difficult to verify the claims of this theory.

8. No Premium

Some researchers argue that there is no equity today and hence there is no equity premium. They interpret "equity premium" in the *ex-ante* sense which is the forward looking measure of the premium i.e. the premium expected to prevail in the future or the conditional equity premium given the current state of the economy.[7]

III Forecasts on the Equity Premium

Since equity-premium realizations are very noisy, forecasting the equity premium with univariate methods is a nearly impossible task. Fortunately, simple economic logic makes predictions about the equity premium, such as high stock prices should imply a low equity premium, the equity premium should usually be positive because of risk aversion, and the cross-sectional pricing of risk should be consistent with the time-series pricing of risk etc. Imposing such economically reasonable guidelines can be of great practical utility in formulating reasonable equity-premium forecasts, but there are wide discrepancies in estimates of the expected equity premium, ranging all the way from 10% to 20%, depending on the source of the forecast.

[7] Source for Models Section: "The Equity Premium, Why is it a Puzzle"

Small investor surveys tend to find equity premium expectations between 10% and 15% per year. In contrast, professionals tend to be more conservative while predicting the equity premium. Individual organizations tend to be in line with professional investors. Financial Engineers appears to use a short-term equity premium of about 6%. The Social Security Administration Office assumes a 4% geometric equity premium, based on a dated historical average. Naturally, those arguing that rescuing Social security requires an asset reallocation into equities contend that the 4% equity premium is too low, based on observed historical averages; others consider this figure too high (Diamond 1999). Studies by Ravi Jagannathan 2000 along with other recent studies by Blachard(1993), Wadhwan (1999), Fama French (2001) were trying to estimate the future return rate of the fixed income asset and the future return rate of the market portfolios. Though different financial models were used in these studies, the agreement has been reached in that the U.S. equity premium is lower than it has been in the history.

However, many financial professors contradicted the view that the equity premium would shrink in the future. According to a comprehensive survey of 226 finance professors, which was conducted by Welch, indicated that on average the finance professors believe that the equity premium in the future would be consistent with the past equity premium with similar level of fluctuation that has been observed historically.

A sampling of finance textbooks shows that, for instance, Copeland, Koller, and Murrin (1995, p. 260) recommends a 5%–6% geometric average. Grinblatt and Titman (1998, p. 174) uses 10% in an example but, after giving a discussion, is silent on giving any estimate. Ross, Westerfield, and Jaffe (1993, p. 257) recommends 8.5%, Van Horne (1992, p. 214) 3%–7%, and Weston, Chung, and Siu (1997, p. 190) 7.5%.

The divergence between the economists and financial professors may be attributable to a few reasons. First, the financial professor projected the future returns by summing the current interest rate on the US treasure bills and the average equity over some historical time period, which was very sensitive to the selected time period. In other words, picking different time period would result in different future return. Therefore it's hard to judge the accuracy of the financial professor's estimation in the survey. Second, there was no sufficient information or historical data provided to correctly predict the future real fixed income return. Jeremy pointed out in his 2000 publication that there were 5% spread between Ibbotson's and Sinquefield's forecast and the actual return in the period of 1976 – 2000 and 1982 -2004. Third, not all financial professors were dedicated in the research area of equity premium. At the time of the survey, they may be more likely to reach their conclusions based on the same reference sources instead of developing and analyzing models by themselves. As a result, bias was embedded in the survey, and it may not be a good representation of the experts focusing on the equity premium study.

As cited in Jeremy's 1999 paper, the real return of the fixed income assets, such as 10- and 30-year TIPS bond yielded 4.0%, and will be much higher in the future than in past century. In addition, it's unlikely that the future growth rates in dividends and earnings are greatly faster than growth rates in GNP, especially in the long run. In other words, the spread between the risk-free rate and the return rate of market portfolios is getting closer and closer. Given the predictions of future micro- and macro-economy situation, and the analysis of various models to calculate the future returns for stock, bonds, and bills, it's more convincing for us to conclude that the equity premium will shrink in the future.

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