One-way ANOVA - details & notation

For example, Layout

1	2	3	4
x x x	x x x	x x x	x x x
x x x	x x x	x x x	x x x

trt. groups

$k = 4$ levels of trt.

$n = 6$ # obs. per cell

Linear Model

$$Y_{ij} = \mu + \tau_i + \epsilon_{ij}$$

jth observation in ith trt. group

$\mu + \tau_i$ = mean of ith group

① $\sum_{i=1}^{k} \tau_i = 0$

② $\epsilon_{ij} \sim \text{Normal}(0, \sigma_\epsilon^2)$

Idea is to partition variation into sources.

$$\bar{Y}_{i\cdot} = \sum_{j=1}^{n} Y_{ij}/n$$ ith cell mean.

$$\bar{Y}_{\cdot\cdot} = \left(\sum_{j=1}^{n} \sum_{i=1}^{k} Y_{ij}\right)/_{k \cdot n}$$

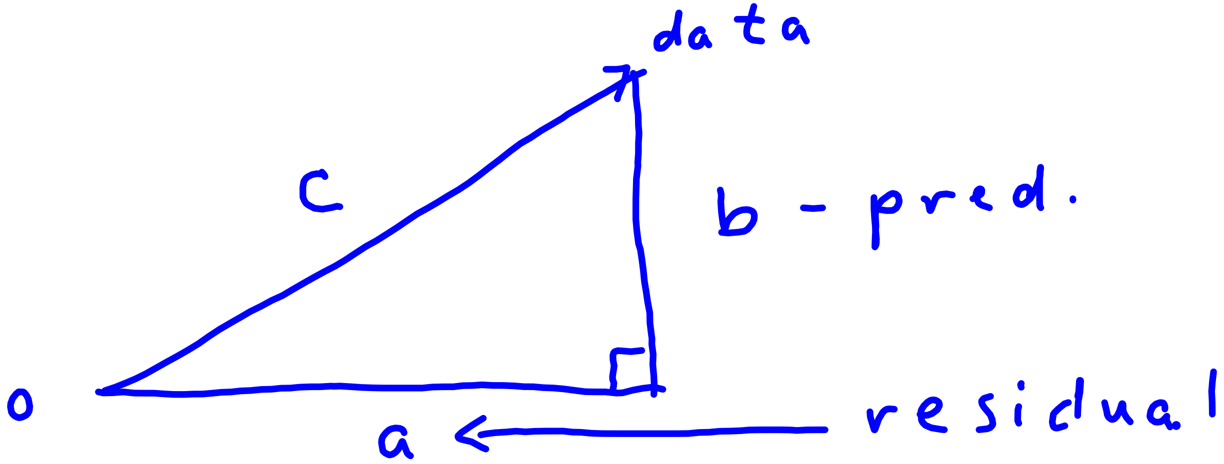
grand mean

$$SS_{total} = \sum_{j=1}^{n} \sum_{i=1}^{k} \left(Y_{ij} - \bar{Y}_{\cdot\cdot}\right)^2$$

$$SS_{trt} = n \cdot \sum_{i=1}^{k} (\bar{Y}_{i.} - \bar{Y}_{..})^2$$

$$SS_{error} = \sum_{j=1}^{n} \sum_{i=1}^{k} (Y_{ij} - \bar{Y}_{i.})^2$$

$$SS_{total} = SS_{trt} + SS_{error}$$



$$c^2 = a^2 + b^2$$

$$\|c\|^2 = \|a\|^2 + \|b\|^2$$

ANOVA table

Source	df	SS	MS
trt.	$K-1$	SS_{trt}	$SS_{trt}/(K-1)$
error	$K(n-1)$	SS_{error}	$SS_{error}/K(n-1)$
	# obs -1		

Hypotheses

test statistic $\left(\frac{\text{signal}}{\text{noise}} \right)$

p-value

conclusion.

$H_0: \mu_i = \mu_j$ all i,j $\iff$ $H_0: \phi_{trt} = 0$

$H_a: \mu_i \neq \mu_j$ some i,j. $\quad$ $H_a: \phi_{trt} > 0$

Recall $\sum \tau_i = 0$

define $\phi_{trt} = \dfrac{\sum_{1}^{K} \tau_i^2}{K-1}$

Math tell us

$$MS_{trt} \sim \sigma_\epsilon^2 + n\phi_{trt}$$

0 or not?

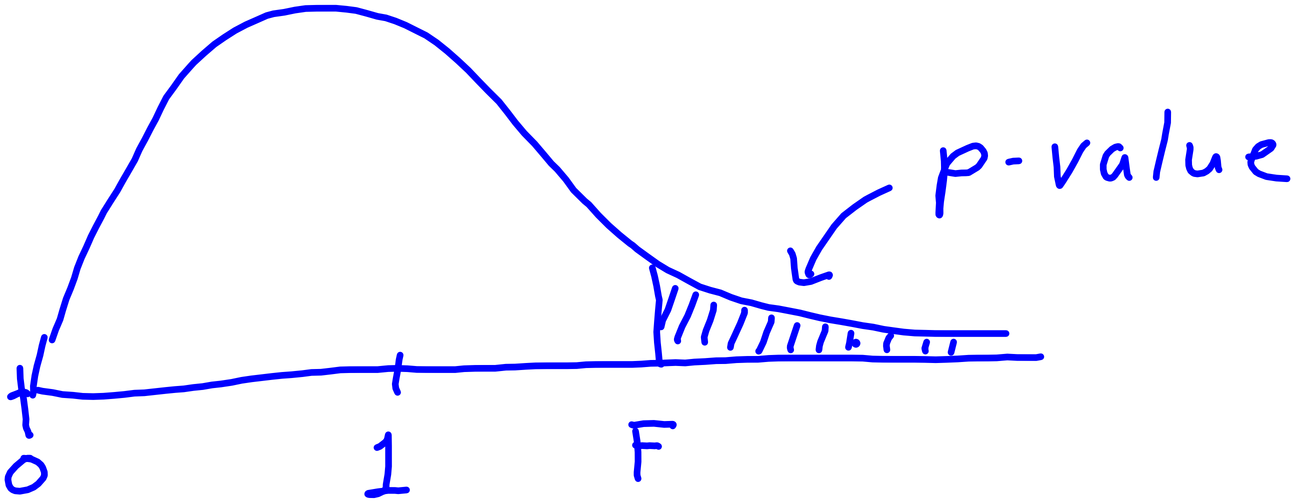
$$MS_{error} \sim \sigma_\epsilon^2$$

$$F_{3,20} = \frac{MS_{trt}}{M_{error}}$$

$(K-1)$ $\quad K(n-1)$

$K=4, \; n=6$

If H_0 true, then test statistic has F-dist'n.



Either

reject H_0 $\Rightarrow$ have evidence treatments differ

or

do not reject H_0 $\Rightarrow$ not enough evidence to reject H_0.

Suppose reject H_0, which trt. means different?

$\mu_1 = \mu_2$ $\mu_2 = \mu_3$ $\mu_3 = \mu_4$

$\mu_1 = \mu_3$ $\mu_2 = \mu_4$ $\frac{K(K-1)}{2} = \frac{4(4-1)}{2}$

$\mu_1 = \mu_4$

$= 6$

comparisons.

Two types of "Multiple Range test".

1. Controls "Comparisonwise error rate"

2. Control "Experimentwise error rate".

Comparisonwise procedure control α for each comparison.

example: Use t-tests at $\alpha = .05$ for each comparison.

critisism: Have 6 tests at $\alpha = .05$, so chance of Type I error higher than .05 over all testing.

Bonferroni -

Comparisonwise $\alpha = .05$

Then $P($any of 6 comparisons false +$)$

$\leq 6(.05) = .30$

Experimentwise error

Comparisonwise procedure:

t-tests

Experimentwise procedure:

Tukey HSD, $\underbrace{\text{SNK}}_{\text{student-Neuman-Keuls}}$

Make 10 comparisons, each at $\alpha = .05$.

The expected # of false positives is

$$10 \times .05 = .5$$

Usual problem

example: radio label