and the $G \times T$ interaction SS are combined: $12.772 + 26.487 = 39.259$ as the teams within groups, which checks with Table 11.8. To get the methods by teams within groups $[MT_{ik(j)}]$ interaction, combine $M \times T$ with $M \times T \times G$ or SS: $5.561 + 5.161 = 10.722$, which is the same as the value in Table 11.8.

The computer printout will also give means computed in all combinations of effects for further analysis if needed.

11.6 REPEATED-MEASURES DESIGN AND NESTED-FACTORIAL EXPERIMENTS

Many statisticians who work with psychologists and educators on the design of their experiments treat repeated-measures designs as a unique topic in these fields of application of statistics. It is the purpose of this section to show that these designs are but a special case of factorial and nested-factorial experiments. Numerical examples and mathematical models are used to illustrate the correspondence between these designs.

One of the simplest of these cases is when a pretest and posttest are given to the same group of subjects after a certain time lapse during which some special instruction may have been administered.

Example 11.3 A recent study at Purdue University was made on a measurement of physical strength (in pounds) given to seven subjects before and after a specified training period. Results are shown in Table 11.13.

Table 11.13 Pretest and Posttest Measures for Example 11.3

Subjects	Pretest	Posttest
1	100	115
2	110	125
3	90	105
4	110	130
5	125	140
6	130	140
7	105	125

Since the same seven subjects were given each test, we have two repeated measures on each subject. Winer [24, p. 266] would handle this as a between-subjects, within-subjects repeated-measures design and report results as shown in Table 11.14.

Table 11.14 ANOVA for Table 11.13

Source of Variation	df		SS		MS	F
Between subjects (S_i)	6		2084.71		347.45	
Within subjects	7		901.00		—	
Tests (T_j)		1		864.29	864.29	145
Residual		6		35.71	5.96	
Totals	13		2985.71			

This analysis implies that the subjects model is

$$Y_{ij} = \mu + S_i + \varepsilon_{j(i)}$$

with df: 6 7

And then the within-subjects' data are further broken down into tests and residual. This residual is actually the subject by test interaction so the model becomes

$$Y_{ij} = \mu + S_i + T_j + ST_{ij}$$

with df: 6 1 6

This model contains no error term since only one pretest score and one post-test score are available on each subject.

Now if this problem were considered in more general terms instead of the so-called repeated-measures format, one would consider this data layout as a two-factor factorial experiment with one observation per treatment. Subjects should be chosen at random and tests should be fixed. Using the algorithm given in Chapter 10, the resulting ANOVA layout would be as shown in Table 11.15.

Here $k = 1$ and a separate error $\varepsilon_{k(ij)}$ is not retrievable. A glance at the EMS column indicates that the only proper F test is to compare the test mean square with the $S \times T$ interaction mean square, which was the test as shown in Table 11.14.

Table 11.15 EMS Determination for Table 11.13

		7	2	1	
		R	F	R	
Source	df	i	j	k	EMS
S_i	6	1	2	1	$\sigma_\varepsilon^2 + 2\sigma_S^2$
T_j	1	7	0	1	$\sigma_\varepsilon^2 + \sigma_{ST}^2 + 7\phi_T$
ST_{ij}	6	1	0	1	$\sigma_\varepsilon^2 + \sigma_{ST}^2$
$\varepsilon_{k(ij)}$	0	1	1	1	σ_ε^2 (not retrievable)

Of course, another approach to this example is to take differences between post- and pretest scores on each individual and test the hypothesis that the true mean of the difference is zero. Using this scheme, a t of 12.05 is found and it is quite well known that this t when squared equals the F of our ANOVA table:

$$t^2 = (12.05)^2 = 145 = F$$

Example 11.4 A slightly more complex problem involves three factors where the subjects are nested within groups. These groups could be classes or experimental conditions and then again each subject is subjected to repeated measures. The study cited on physical strength was extended so that three groups of subjects were involved, two being subjected to special experimental training and the third acting as a control with no special training. Again each subject was given a pre- and posttest where the measurement here was velocity of a baseball throw in meters per second. The resulting data are given in Table 11.16. If these data are treated as a

Table 11.16 Pretest and Posttest Throwing Velocities of Three Groups of Subjects in Meters/Second

Group	Subject	Pretest	Posttest
	1	26.25	29.50
	2	24.33	27.62
	3	22.52	25.71
I	4	29.33	31.55
	5	28.90	31.35
	6	25.13	29.07
	7	29.33	31.15
	8	27.47	28.74
	9	25.19	26.11
	10	23.53	25.45
II	11	24.57	25.58
	12	26.88	27.70
	13	27.86	28.82
	14	28.09	28.99
	15	22.27	22.52
	16	21.55	21.79
	17	23.31	23.53
III	18	30.03	30.21
	19	28.17	28.65
	20	28.09	28.33
	21	27.55	27.86

Table 11.17 ANOVA of Table 11.16

Source	df		SS		MS	EMS	F
Between subjects	20		271.05		—		
Groups (G_i)		2		28.14	14.07	$\sigma_e^2 + 2\sigma_S^2 + 14\phi_G$	1.04
Subjects within groups ($S_{j(i)}$)		18		242.91	13.50	$\sigma_e^2 + 2\sigma_S^2$	
Within subjects	21		35.73		—		
Tests (T_k)		1		21.26	21.26	$\sigma_e^2 + \sigma_{TS}^2 + 21\phi_T$	183***
$G \times T$		2		12.38	6.19	$\sigma_e^2 + \sigma_{TS}^2 + 7\phi_{GT}$	53**
$T \times S_{kj(i)}$		18		2.09	0.116	$\sigma_e^2 + \sigma_{TS}^2$	

repeated-measures design, the results are usually presented as in Winer [24, p. 520], here shown as Table 11.17.

If the layout of Table 11.16 is treated as a nested-factorial experiment where subjects are nested within groups and then tests are factorial on both groups and subjects, the model can be written as follows:

$$Y_{ijk} = \mu + G_i + S_{j(i)} + T_k + GT_{ik} + TS_{kj(i)} + \varepsilon_{m(ijk)}$$

with df: 2 18 1 2 18 0

including a nonretrievable error term. Use of this model and the algorithm cited above gives Table 11.18.

Table 11.18 EMS Determination for Table 11.16

		F	R	F	R	
		3	7	2	1	
Source	df	i	j	k	m	EMS
G_i	2	0	7	2	1	$\sigma_e^2 + 2\sigma_S^2 + 14\phi_G$
$S_{j(i)}$	18	1	1	2	1	$\sigma_e^2 + 2\sigma_S^2$
T_k	1	3	7	0	1	$\sigma_e^2 + \sigma_{TS}^2 + 21\phi_T$
GT_{ik}	2	0	7	0	1	$\sigma_e^2 + \sigma_{TS}^2 + 7\phi_{GT}$
$TS_{kj(i)}$	18	1	1	0	1	$\sigma_e^2 + \sigma_{TS}^2$
$\varepsilon_{m(ijk)}$	0	1	1	1	1	σ_e^2 (not retrievable)

This scheme generates the EMS column that was simply reported in Table 11.17, and that column indicates the proper tests to be made. The numerical results are, of course, the same.

The nested-factorial experiment covers many situations in addition to the repeated-measures design. The factor in the nest may be farms within