16.6 RANDOMIZED INCOMPLETE BLOCKS—RESTRICTION ON EXPERIMENTATION

Method for Balanced Blocks

In some randomized block designs it may not be possible to apply all treatments in every block. If there were, for example, six brands of tires to test, only four could be tried on a given car (not using the spare), and such a block would be incomplete, having only four out of the six treatments in it.

Example 16.6 Take the problem of determining the effect on current flow of four treatments applied to the coils of TV tube filaments. As each treatment application requires some time, it is not possible to run several observations of these treatments in one day. If days are taken as blocks, all four treatments must be run in random order on each of several days in order to have a randomized block design. After checking it is found that even four treatments cannot be completed in a day; three are the most that can be run. The question then is: Which treatments are to be run on the first day, which on the second, and so forth, if information is desired on all four treatments?

The solution to this problem is to use a balanced incomplete block design. An *incomplete block design* is simply one in which there are more treatments than can be put in a single block. A *balanced incomplete block design* is an incomplete block design in which every pair of treatments occurs the same number of times in the experiment. Tables of such designs may be found in Fisher and Yates [11]. The number of blocks necessary for balancing will depend upon the number of treatments that can be run in a single block.

For the example mentioned there are four treatments and only three treatments can be run in a block. The balanced design for this problem requires four blocks (days) as shown in Table 16.18.

Table 16.18 Balanced Incomplete Block Design for TV Filament Example

Block (days)	Treatment A	B	C	D	$T_{i.}$
1	2	—	20	7	29
2	—	32	14	3	49
3	4	13	31	—	48
4	0	23	—	11	34
$T_{.j}$	6	68	65	21	$160 = T_{..}$

In this design only treatments A, C, and D are run on the first day; B, C, and D on the second day, and so forth. Note that each pair of treatments, such as AB, occurs together twice in the experiment. A and B occur together on days 3 and 4; C and D occur together on days 1 and 2; and so on. As in randomized complete block designs, the order in which the three treatments are run on a given day is completely randomized.

The analysis for this design is easier if some new notation is intro-

Table 16.20 ANOVA for Incomplete Block Design Example for Both Treatments and Blocks

Source	df	SS	MS
Blocks (adjusted)	3	6.17	2.06
Blocks	(3)	(100.67)	—
Treatments (adjusted)	3	880.83	293.61
Treatments	(3)	(975.34)	—
Error	5	363.17	72.63
Totals	11	1344.67	

It should be noted also that the final sum of squares values used in Table 16.20 to get the mean square values do not add up to the total sum of squares. This is characteristic of a nonorthogonal design. The F test for blocks was not run because its value is obviously extremely small, which indicates no day-to-day effect on current flow.