Taguchi Methods
and
Response Surface
Alternatives
Once in control, goal is to produce uniformly near a target.

- Mean on target
- Reduce variation
Three possible situations:

- Smaller is better
- Larger is better
- On target is best
On target is best.
Taguchi considered deviation from target to be loss to society. Taguchi models this as:

\[ L(Y) = K(Y - T)^2 \]

\[ Y = \text{Quality Control} \]

\[ T = \text{Target value} \]
On average, loss is proportional to:

$$E(Y - T)^2$$

$$= Var(Y)$$

$$+ (Bias)^2$$

where

$$Bias = \text{difference of mean and Target}$$
For smaller is better, Taguchi has loss as:

\[ L(Y) = Y^2 \]

i.e. on average

\[ E(Y^2) \]

So Target is 0.
For larger is better, Taguchi considers loss as:

\[ L(Y) = 1/y^2 \]

or

\[ \text{E}(1/y^2) \].
First step in Taguchi Method is parameter design. Parameters are inputs known or suspected to affect the quality characteristic.

Goal: Design in quality, don’t sample to catch poor output.
Parameters are divided into two groups:

- Control variables - parameters which can be controlled in the process.
- Noise variables - parameters which vary and are usually not controlled (sometimes hard or expensive to control).
Levels are chosen for each of the control variables.

Levels are chosen and temporarily fixed for each of the noise variables.
Control variables are placed into inner array. Example:

\[
\begin{bmatrix}
1 & 1 & 1 \\
1 & 1 & 2 \\
1 & 2 & 1 \\
1 & 2 & 2 \\
2 & 1 & 1 \\
2 & 1 & 2 \\
2 & 2 & 1 \\
2 & 2 & 2 \\
\end{bmatrix}
\]

Eight design points, each row one design point
Noise variables are placed in outer array

Example:

\[
\begin{bmatrix}
1 & 1 \\
1 & 2 \\
2 & 1 \\
2 & 2
\end{bmatrix}
\]

Generally the levels span what is experienced in production conditions.
The structure of each array is typically

- Factorial design
- Fractional factorial design (sometimes referred to as "orthogonal array").

The arrays are then crossed and observations collected.
Goal of analysis: Minimize

\[ E(Y - T)^2 \]

by choice of best control variable settings.

Taguchi suggests look at "signal to noise" ratio. Bigger is better. Then adjust mean.
Criticisms:

• Crossing inner and outer arrays can lead to unwieldy designs.

• Use of signal to noise ratios very ad-hoc and hard to generalize (various versions).
Alternatives proposed include Response Surface Methodology (RSM).

Goals:

- Model the response $Y$ as a function of control and noise variables.
- Use information on distribution of noise variables to model mean and variance.
Simple example:

\[ Y = \text{Lamina thickness} \]
\[ X_1 = \text{Viscosity (control variable)} \]
\[ X_2 = \text{Ambient humidity (noise variable)} \]

Fitted model:

\[ \hat{Y} = \hat{\beta}_0 + \hat{\beta}_1 X_1 + \hat{\beta}_{12} X_1 X_2 \]

Select \( X_1 \) to minimize loss.
\[ E(\hat{Y}) = \hat{\beta}_0 + \hat{\beta}_1 X_1 + \hat{\beta}_{12} X_1 \cdot E(X_2) \]

\[ Var(\hat{Y}) = (\hat{\beta}_{12} X_1)^2 \cdot Var(X_2) + \sigma^2 \]

(Conditional mean and variance, conditional on parameter estimates).
More complex example:

\[ X_1, X_2 \text{ -- Control} \]
\[ X_3 \text{ -- Noise} \]
\[ Y = \beta_0 + \hat{\beta}_1 X_1 + \hat{\beta}_2 X_2 \]
\[ + \hat{\beta}_{23} X_2 \cdot X_3 \]

\[ E(\hat{Y}) = \beta_0 + \hat{\beta}_1 X_1 + \hat{\beta}_2 X_2 + \hat{\beta}_{23} X_2 \cdot (EX_3) \]
\[ Var(\hat{Y}) = (\hat{\beta}_{23} X_2) \cdot Var(X_3) + \sigma \]

Can set \( X_2 \) to minimize variance

set \( X_1 \) to control mean.