STAT553 HW07 **KEY**

1. The Working-Hotelling confidence band is

$$y = \bar{Y} + \hat{\beta}_1(x - \bar{x}) \pm \lambda S \sqrt{\frac{1}{n} + \frac{(x - \bar{x})^2}{\sum(x_i - \bar{x})^2}}$$

   where $\lambda = (2F^{\alpha}_{2,m-2})^{1/2}$, and similarly for the second regression line. By replacing α with $\alpha/2$ in the quantile, we get the Bonferroni simultaneous confidence band for both regression lines.

   In fact, knowing that the two parts are independent, we can do better than Bonferroni. Replace α with $1 - \sqrt{1-\alpha}$ in the quantiles, then

$$P(\text{First band covers}) = \sqrt{1-\alpha}$$
$$P(\text{Both bands cover}) = \sqrt{1-\alpha} \cdot \sqrt{1-\alpha} = 1 - \alpha$$

   This is slightly narrower than Bonferroni.

2. (a) The full model has 4 degrees of freedom and the reduced model has 1 degree of freedom. The simultaneous confidence interval is

$$h'\mu = h'\hat{\mu} \pm (3F^{\alpha}_{3,4n-4} S^2 \frac{h'h}{n})^{1/2}$$

   for all $h \in B_1$ where

$$B_1 = \text{span}\{(1,1,0,0)', (1,0,-1,0)', (1,0,0,1)'\}$$

   (b) Tukey's simultaneous confidence interval applies only to contrasts, thus

$$B_2 = B_1 \cap \{\text{All contrasts}\} = \text{span}\{(1,0,-1,0)', (0,1,0,-1)'\}$$

   The simultaneous confidence interval is

$$h'\mu = h'\hat{\mu} \pm q^{(\alpha)}_{4,4n-4} \frac{S}{\sqrt{n}} \sum_{i=1}^{4} \frac{|h_i|}{2}$$