STAT553 HW05 **KEY**

Ex3f-1. Calculation depends on result of theorem 3.6.

$$\begin{aligned}
& Y'RY - Y'R_GY \\
=& Y'RY - Y'(Y - X\hat{\beta}_G - Z\hat{\gamma}_G) \\
=& Y'RY - Y'(Y - X\hat{\beta} + XL\hat{\gamma}_G - Z\hat{\gamma}_G) \\
=& Y'RY - Y'RY + Y'(-XL + Z)\hat{\gamma}_G \\
=& Y'RZ\hat{\gamma}_G \qquad (1) \\
& \sigma^2\hat{\gamma}_G'(Var(\hat{\gamma}_G))^{-1}\hat{\gamma}_G \\
=& \sigma^2\hat{\gamma}_G \quad \frac{1}{\sigma^2}Z'RZ \quad \hat{\gamma}_G \\
=& \hat{\gamma}_G'Z'RZ \quad (Z'RZ)^{-1}Z'RY \\
=& \hat{\gamma}_G'Z'RY
\end{aligned}$$

Ex3f-2.

$$\begin{aligned}
& \frac{\partial}{\partial\gamma}(Y - Z\gamma)'R(Y - Z\gamma) \\
=& \frac{\partial}{\partial\gamma}(Y'RY - 2\gamma'Z'RY + \gamma'Z'RZ\gamma) \\
=& -2Z'RY + 2Z'RZ\gamma
\end{aligned}$$

Let it equal to 0 we get $\gamma_{minimizer} = \hat{\gamma}_G = (Z'RZ)^{-1}Z'RY$. Plug this in,

$$\begin{aligned}
& (Y - Z\hat{\gamma}_G)'R(Y - Z\hat{\gamma}_G) \\
=& Y'RY - 2\hat{\gamma}_G'Z'RY + \hat{\gamma}_G'Z'RZ\hat{\gamma}_G \\
=& Y'RY - \hat{\gamma}_G'Z'RY \\
=& Y'R_GY
\end{aligned}$$

Ex3f-3.

$$\begin{aligned}
Var(\hat{\beta}) &= \sigma^2(X'X)^{-1} \\
Var(\hat{\beta}_G) &= \sigma^2(X'X)^{-1} + LML'
\end{aligned}$$

Since M is semi-positive definite, so does LML'. So the (i,i) element of it is non-negative, thus $Var(\hat{\beta}_{G,j}) \geq Var(\hat{\beta}_j)$.