

1. Prove Schwartz equality: for a positive definite matrix  $A$ ,

$$\sup_{a \neq 0} \frac{(a'c)^2}{a' A a} = c' A^{-1} c$$

with equality if and only if  $a$  is proportional to  $A^{-1}c$ .

2. Prove that  $(1 - \alpha/k)^k > (1 - \alpha)$ , for  $0 < \alpha < 1$  and  $k > 1$ . Use this to explain that Bonferroni method is conservative for cases when we test  $k$  independent test simultaneously.
3. Use the Working-Hotelling confidence band as an example, explain the difference between the concept of confidence interval, and simultaneous confidence band.
4. Consider the problem with  $K$  simple lines with observations  $Y_{kj} = \alpha_k + \beta_k x_{kj}$ ,  $j = 1, \dots, n_k$ ,  $k = 1, \dots, K$ .
- Find the estimate of  $\beta$  if  $\beta_1 = \dots = \beta_K = \beta$ .
  - Find the estimate of  $\alpha$  if  $\alpha_1 = \dots = \alpha_K = \alpha$
  - What is the test statistic for testing the hypothesis  $H : \alpha_1 = \dots = \alpha_K$

5. Consider a model

$$Y_{ijk} = \mu + \alpha_i + \beta_j + \beta x_{ij} + \varepsilon_{ijk}$$

for  $i = 1, \dots, I$ ,  $j = 1, \dots, J$ ,  $k = 1, \dots, K$ ; where  $\sum \alpha_i = 0 = \sum \beta_j$ . Find the LSE of  $\mu, \alpha, \beta_i$  and  $\beta$

6. Seber Miscellaneous exercise 5, #1.