Statistics 514: Design of Experiments

Topic 6 Summary

One-factor Experiments

- Generalizes 2-sample t-test
- Two equivalent models: cell means/treatment effects

$$\begin{aligned} Y_{i,j} &= \mu_i + e_{i,j} \\ Y_{i,j} &= \mu + \tau_i + e_{i,j}, \ \sum \tau_i = 0 \end{aligned}$$

- ANOVA
  - Decompose total variation into treatment and chance variation.
  - *Null model:* no treatment contribution
  - *Alternative model:* F-ratio is big (compared to 1, depending on n and $\sum \tau_i^2$).
  - Summarized in ANOVA table.
- Equivalent to general linear tests in regression with indicator variables.
- Estimates are LS estimates.
- Unbalanced design – affects expectation of SS_{TRT}

ANOVA

- Expressed in terms of
  - regression (computation and generalization)
  - contrasts (more specific questions)
- Can be decomposed into (orthogonal) contrasts.
- Effects can be *fixed* or *random*, depending on whether levels are all used or sampled.

Contrasts

- Linear combinations with coefficients summing to zero
  - Can have direct meaning
  - (Often) get more power with using common MSE
- Test using t-test or (more generally) F-tests

- *Orthogonality* – inner product (downweighted by sample size) sums to zero
  - There are at most $a-1$ (pairwise) orthogonal contrasts in a set.
  - Orthogonal contrasts of sample means are independent (under normality).
- SS_{Trt} is a sum of $a-1$ orthogonal contrast sums of squares

Random Effects

- Inference on (randomly) unseen levels ($\mathrm{H}_0 : \sigma_\tau^2 = 0$)
- Similar models ($\tau_i \sim N(0, \sigma_\tau^2)$), same inference
- Interested in estimating/bounding variance components (sources of variability, e.g., σ_τ^2)
- Unbiased (ANOVA/`type1`) estimates can be negative.
- SAS
  - `random` option in `proc glm` (for specifying random components; gives expected mean squares)
  - `proc varcomp` (for estimating variance components)
  - `proc mixed` (for getting nonnegative ML estimates)