

Statistics 514: Design of Experiments

Topic 3 Summary

Two-sample t -test

- (Null) Hypothesis to test: $\mu_1 = \mu_2$
- Data: $\{y_{1,1}, \dots, y_{1,n_1}, y_{2,1}, \dots, y_{2,n_2}\}$.
- Model:

$$\begin{aligned}y_{i,j} &= \mu + \tau_i + \epsilon_{i,j} \\i &= 1, 2 \quad j = 1, \dots, n_i \\ \epsilon_{i,j} &\sim N(0, \sigma^2)\end{aligned}$$

- Model Hypothesis: $\tau_1 = \tau_2 = 0$
- Test Statistic:

$$\begin{aligned}t_0 &= (\bar{y}_1 - \bar{y}_2) / S_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}} \sim t_{n_1+n_2-2} \\ S_p^2 &= \frac{(n_1 - 1)S_1^2 + (n_2 - 1)S_2^2}{n_1 + n_2 - 2}\end{aligned}$$

- If t_0 is larger than $(1 - \alpha/2)\%$ quantile, reject.
- Assumptions
 - Normality
 - Constant Variance
 - Independence

Two-sample t -test: Alternatives

- Alternatives don't require so many assumptions.
 - Typically less powerful if assumptions met.
 - Paired t -test: exploits design structure which leads to positive correlation between groups in data; simpler test, more complicated model.

$$y_{i,j} = \mu + \tau_i + \beta_j + \epsilon_{i,j}$$

Power and Sample Size

- *Goal:* maximize or bound power ($1 - P(\text{Type II})$, probability of correctly rejecting null hypothesis) given limits on $P(\text{Type I})$.
 - Increasing power (like reducing variance or bias) is often the motivating factor in many aspects of statistical practice.
- *Steps:*
 - Find rejection region (under null hypothesis)
 - Find probability of being in rejection region (under alternative hypothesis)
- Requires knowing (or guessing) α , δ , σ , and n .
- Uses non-central distributions.
- Often change n until desired power achieved.

Confidence Intervals

- Like doing lots of hypothesis tests at once.
- Covers true value (at least) $1 - \alpha$ of the time.
- *Usual form:* $\hat{\theta} \pm t\sqrt{\text{Var}(\hat{\theta})}$
 - Sometimes based on quantiles of test statistic.
 - Manipulate expression until parameter is isolated (in the middle).
- Given notion of accuracy *after* data is collected.
- Reject $H_0 : \theta = \theta_0$ if θ_0 not in interval