Statistics 514: Problem Set No. 6

Due Thursday, November 15/Session 25

1. Reanalyze Problem 9 of Problem Set 5 as a replicated Latin square (so, two Latin squares side-by-side, not a Latin rectangle) where rows are repeated but columns are not.

2. Pollutants may reduce the strength of bird bones. We believe that the strength reduction, if present, is due to a change in the bone itself, and not a change in the size of the bone. One measure of bone strength is calcium content. We have an instrument which can measure the total amount of calcium in a 1 cm length of bone. Bird bones are essentially thin tubes in shape, so the total amount of calcium will also depend on the diameter of the bone.

   Thirty-two chicks are divided at random into four groups. Group 1 is a control group and receives a normal diet. Each other group receives a diet including a different toxin (pesticides related to DDT). At 6 weeks, the chicks are sacrificed and the calcium content (in mg) and diameter (in mm) of the right femur is measured for each chick.

Control		P #1		P #2		P #3	
C	Dia	C	Dia	C	Dia	C	Dia
10.41	2.48	12.10	3.10	10.33	2.57	10.46	2.6
11.82	2.81	10.38	2.61	10.03	2.48	8.64	2.17
11.58	2.73	10.08	2.49	11.13	2.77	10.48	2.64
11.14	2.67	10.71	2.69	8.99	2.30	9.32	2.35
12.05	2.90	9.82	2.43	10.06	2.56	11.54	2.89
10.45	2.45	10.12	2.52	8.73	2.18	9.48	2.38
11.39	2.69	10.16	2.54	10.66	2.65	10.08	2.55
12.5	2.94	10.14	2.55	11.03	2.73	9.12	2.29

   Analyze these data with respect to the effect of pesticide on calcium in bones.

3. The following table represent a hypothetical three-group study assessing different interventions for depression. Specifically, suppose 30 depressive individuals have been randomly assigned to one of three conditions: (1) selective serotonin reuptake inhibitor (SSRI) antidepressant medication, (2) placebo, or (3) wait list control. The Beck Depression Inventory (BDI) has been administered to each individual prior to the study, and then is later administered a second time at the end of the study. In essence, the question of interest is whether individuals in some groups change more than do individuals in other groups.

SSRI		Placebo		Wait list	
Pre	Post	Pre	Post	Pre	Post
18	12	18	11	15	17
16	0	16	4	19	25
16	10	15	19	10	10
15	9	14	15	29	22
14	0	20	3	24	23
20	11	25	14	15	10
14	2	11	10	9	2
21	4	25	16	18	10
25	15	11	10	22	14
11	10	22	20	13	7

(a) Perform an ANCOVA analysis, testing if there is a significant difference between groups on the posttest.

(b) Because groups were randomly assigned here, it might seem acceptable to ignore pretest scores in comparing groups on the posttest. Test to see whether there is a significant difference between groups in the pretest.

(c) Based on your answer to the previous question, you might decide to proceed to conduct ANOVA on posttest scores, ignoring pretest scores. How do your results compare to the ANCOVA results? In particular, does the ANOVA yield a statistically significant result at the 0.05 level?

(d) Another possible approach might be to analyze the differences between the scores. Conduct an ANOVA on the "gain scores". How do your results compare to the ANCOVA results? In particular, does the ANOVA yield a statistically significant result at the 0.05 level?

(e) What do your results comparing the three different approaches suggest about the relative merits of ANCOVA, ANOVA on posttest only, and ANOVA on gain scores for analyzing data from randomized pretest-posttest designs?

4. This problem asks you to compare the confidence intervals obtained from each of the same three methods of analysis used in the last problem (on the same data). Specifically, assume throughout the problem that simultaneous 95% confidence intervals are of interest for the collection of three pairwise comparisons. (Use an appropriate method for controlling familywise error rate.)

   (a) Begin by forming 95% simultaneous confidence intervals for each pairwise difference in posttest means, adjusted via ANCOVA.

   (b) Because intervals were randomly assigned to groups, you might decide to proceed to form confidence intervals based on an ANOVA on posttest scores, ignoring pretest scores. In other words, you decide to form 95% simultaneous confidence intervals for each pairwise difference in posttest means. How do your intervals compare to the ANCOVA intervals in the previous problem?

   (c) Another possible approach might be to analyze the differences between pretest and posttest scores. Form 95% simultaneous confidence intervals for each pairwise difference in gain score means. How do your results compare to the ANCOVA intervals?

   (d) What do your results comparing the three different approaches suggest about the relative merits of ANCOVA, ANOVA on posttest only, and ANOVA on gain scores for analyzing data from randomized pretest-posttest designs?

5. A study has been conducted on the environmental impact of an industrial incinerator. One of the concerns is the emission of heavy metals from the stack, and one way to measure the impact is by looking at metal accumulations in soil and seeing if nearby sites have more metals than distant sites (presumably due to deposition of metals from the incinerator).

   Eleven sites of one hectare each (100m by 100m) were selected around the incinerator. Five sites are of agricultural soils, while the other six are on forested soils. Five of the sites were located near the incinerator (on their respective soil types), while the other sites were located far from the incinerator. At each site, nine locations are randomly selected within the site and mineral soil sampled at each location. We then measure the mercury content in each sample (mg/kg).

Complicating any comparison is the fact that heavy metals are generally held in the organic portion of the soil, so that a soil sample with more carbon will tend to have more heavy metals than a sample with less carbon, regardless of the deposition histories of the samples, soil type, etc. For this reason, we also measure the carbon fraction of each sample (literally the fraction of the soil sample that was carbon).

The data given below are site averages for carbon and mercury. Analyze these data to determine if there is any evidence of an incinerator effect on soil mercury.

Soil	Distance	Carbon	Mercury
Agricultural	Near	0.0084	0.0128
Agricultural	Near	0.0120	0.0146
Agricultural	Near	0.0075	0.0130
Agricultural	Far	0.0087	0.0133
Agricultural	Far	0.0105	0.0090
Forest	Near	0.0486	0.0507
Forest	Near	0.0410	0.0477
Forest	Far	0.0370	0.0410
Forest	Far	0.0711	0.0613
Forest	Far	0.0358	0.0388
Forest	Far	0.0459	0.0466

6. The data given in the set `PS6prob6` compares the effects of heroin, morphine, and a placebo on the level of mental activity for human subjects. While a strong linear relationship is indicated between the before and after values for the placebo data, the data for morphine and heroin have a covariance adjustment slope that is essentially 0, as though there is no linear relationship. How can you make sense of this apparent contradiction? How would change your model to analyze these data? Report the results of your analysis.

7. To determine the performance of an engine, its throttle was measured at various combinations of speed and torque. The resultant data are given in data file PS6prob7.dat with columns corresponding to speed (rpm), torque (ft-lbs), and coded throttle (volts). Fit a second-order model to these 134 observations, and present your results. In addition, determine the combination of engine speed and torque that produces the lowest throttle value.

8. Millers want to make bread flours that bake into large loaves. They need to mix flours from four varieties of wheat, so they run an experiment with different mixtures and measure the volume of the resulting leaves (ml/100g dough). The experiment was performed on 2 separate days, obtaining the following results (data from Draper *et al* 1993):

Day 1					Day 2				
x_1	x_2	x_3	x_4	Volume	x_1	x_2	x_3	x_4	Volume
0	0.25	0	0.75	403	0	0.75	0	0.25	423
0.25	0	0.75	0	425	0.25	0	0.75	0	417
0	0.75	0	0.25	442	0	0.25	0	0.75	388
0.75	0	0.25	0	433	0.75	0	0.25	0	407
0	0.75	0.25	0	445	0	0	0.25	0.75	338
0.25	0	0	0.75	435	0.25	0.75	0	0	435
0	0	0.75	0.25	385	0	0.25	0.75	0	379
0.75	0.75	0	0	425	0.75	0	0	0.25	406
0.25	0.25	0.25	0.25	433	0.25	0.25	0.25	0.25	439

Analyze these data to determine which mixture of flours yields the largest loaves.

9. The data set `PS6prob9.dat` is the result of a three factor factorial design, where all three factors are considered random.

   (a) Perform the appropriate analysis (using an F-test based on computing the expected mean squares). If an exact F-test does not exist, use Satterthwaite's approximate F-test procedure.

   (b) Estimate all variance components using the ANOVA method.

   (c) For any component with a positive estimate, compute an approximate 90% CI using Satterthwaite's method.

10. Assume you are hoping to submit a grant proposal to investigate dose-response relationships in a psychopharmacological study of depression, planning to pursue your work in New Mexico, where psychologists were recently granted prescription privileges. From the range of acceptable dosages of your favorite antidepressant, you select three dosages at random for investigation. Four psychologists are available for the study, and you regard them as representative of the pool of psychologists who might eventually be granted prescription privileges. Each psychologist in your pilot study prescribes each of the three dosage levels to five of his or her depressed clients. When scores on a depression scale are gathered at the end of the study and analyzed, the following mean squares are obtained, with the associated degrees of freedom shown in the following table.

Effect	MS	df
drug	3.0	2
psychologist	2.0	3
drug $\times$ psychologist	1.0	6
error	1.0	48

   (a) Carry out appropriate tests of both the main effects and interaction.

   (b) Using the mean squares given, compute estimates of the variance components associated with the two main effects, the interaction, and within-group error.

   (c) Use the variance components computed in the previous section to estimate values of the intraclass correlation for each of the effects in the design.

   (d) Assume in your grant proposal you want to project the sample size required to achieve a particular level of power for detecting an effect of the drug dosage factor. As a first step, how could you express algebraically the ratio of the mean squares of the numerator and denominator terms that would be involved in the test of this factor? How could this ratio be expressed if you were to substitute the numerical values of your variance components as estimate of the variances in the expected mean squares?

   (e) Assuming you were to continue with a sample of four psychologists in your study, how many subjects would each of these have to treat in order to achieve of power of 0.80 for detecting the drug effect at an α of 0.05? If you were able to recruit a total of eight psychologists to participate in your study, how many subjects would each need to treat to achieve a power of 0.80?