

**Statistics 514: Problem Set No. 5**  
Due Thursday, November 1/Session 21

1. An experiment was conducted to determine the effect of germination time (in days) and temperature (degrees C) on the free alpha amino nitrogen (FAN) content of rice malt. The value shown in the following are the treatment means of FAN with  $n = 2$  (data from Aniche Okafor 1989).

Days	Temperature				Row Means
	22	24	26	28	
1	39.4	49.9	55.1	59.5	50.98
2	56.4	68.0	76.4	88.8	72.40
3	70.2	81.5	95.6	99.6	86.72
Column Means	55.33	66.47	75.70	82.63	
Grand Mean	70.03				

The total sum of squares is 8097. Draw an interaction plot for these data. Compute an ANOVA table.

2. Consider Montgomery Problem 5-2.
  - (a) Construct an interaction plot and determine the standard error of each mean in the plot.
  - (b) Comment on what the interaction plot tells you.
  - (c) Analyze the data using  $\alpha = 0.05$ .
3. Set out an orthogonal partition of  $6 \times 6$  Latin squares in which a second alphabet is superimposed on the first with two letters each occurring three times in each row and column and three times coincident with each letter of the Latin square. Find also an arrangement with three letters each occurring twice in each row, etc. Give the form of the analysis of variance in each case. These partitions in the  $6 \times 6$  were given by Finney (1945) as in one sense the nearest one could get to a  $6 \times 6$  Graeco-Latin square.
4. Raghavarao and Zhou have studied incomplete block designs in which each triple of treatments occur together in a block the same number of times. One application is to marketing studies in which  $a$  versions of a commodity are to be compared, and each shop is to hold only  $k < a$  versions. The design ensures that each version is seen in comparison with each pair of other possibilities the same number of times. Give such a design for  $a = 6$ ,  $k = 4$ , and suggest how the responses might be analyzed.
5. In 1850 the Reverend T. P. Kirkman posed the following problem. A schoolmistress has a class of 15 girls and wishes to take them on a walk each weekday for a week. The girls should walk in threes with no two girls together in a triplet more than once in a week. Show that this is equivalent to finding a balanced incomplete block design and given an explicit solution. (For partial credit, given an explicit solution to the situation where there are 12 girls in groups of three. How many days are necessary?)

**Note:** Keep in mind that many design examples are recorded in books and on the Web. So fabricating the solution on your own might not be very efficient with these resources available.

6. In an experiment on automobile mileages, four test drivers traveled 1000 miles on each of four brands of automobile (A, B, C, D). The speed varied from 25 to 60 miles per hour. Averages of the mileages are presented in the following table. For this example, the automobile brands are the treatments, and the test drivers are the blocks.

		Automobile brands			
		A	B	C	D
Test Driver	1	22		32	26
	2	28	26	34	
	3	22	20		24
	4		22	34	26

- (a) Find the intrablock estimate for the difference of the effects of the mileages for automobile brands A and B and estimate its standard error.
- (b) Find the combined intra and inter block estimate for the difference in the mileages for A and B and estimate its standard error.
7. We saw in class that for a BIBD,  $\text{Var}(\hat{\tau}_j - \hat{\tau}_i) = (2k/\lambda a)\sigma^2$ . If the experiment were run as a randomized complete block,  $\text{Var}(\hat{\tau}_j - \hat{\tau}_i)$  would equal  $\text{Var}(\bar{y}_j - \bar{y}_i) = 2\sigma^2/b$ . Prove that  $2k/\lambda a > 2/b$  and give it a statistical interpretation.
- 8.
9. The data below are from a replicated Latin Square with four treatments and 32 experimental units. Test for treatment differences, and use Tukey HSD with level 0.01 to analyze the pairwise treatment differences.

D	44	B	26	C	67	A	77	B	51	D	62	A	71	C	49
C	39	A	45	D	71	B	74	C	63	A	74	D	67	B	47
B	52	D	49	A	81	C	88	A	74	C	75	B	60	D	58
A	73	C	58	B	76	D	100	D	82	B	79	C	74	A	68