Statistics 514: Problem Set No. 4

Due Thursday, October 18/Session 17

1. You want to compare 3 treatments in a one-way fixed effects ANOVA. If the $a = 3$ treatment means were as different as $100 - \Delta$, 100, $100 + \Delta$, you'd want at least an 80% chance of detecting at the 5% level of significance. Suppose that $\Delta = 5$ and $\sigma = 10$. How large must n be?

   You can follow the pattern in Example 3-10 of Montgomery. As in the example, it is probably a good idea to record your calculations for several guesses at n. This lets you check that increasing n increases the probability of rejecting the hypothesis. It also lets you see the effects of change n.

   Hint: If you use the table in Montgomery, photocopy and enlarge it. At the same time, using SAS would probably be faster for this and the next problem.

   The book *How Many Subjects?* by Kraemer and Thiemann gives a cookbook approach to picking the sample size that is easier to implement than these tables. (It also offers up some good advice borne of experiment with human subjects for those designing such studies, and sample size determinations for lots of problems.) The book *Statistical Power Analysis for the Behavioral Sciences (Second edition)* by Cohen offers more discussion on power calculations.

2. Suppose that an experiment will use $n = 4$ observations from each group. (In other words, rather than knowing Δ and σ as in the previous problem, you know n.) With 5% significance and an 80% chance of rejecting the null, how small a difference Δ/σ can this experiment detect?

3. We are planning an experiment on the quality of video tape and have purchased 24 tapes, four tapes from each of six types. The six types of tape were

   (a) brand A high cost,
   (b) brand A low cost,
   (c) brand B high cost
   (d) brand B low cost,
   (e) brand C high cost,
   (f) brand D high cost.

   Each tape will be recorded with a series of standard test patterns, replayed 10 times, and then replayed an eleventh time into a device that measures the distortion on the tape. The distortion measure is the response, and the tapes will be recorded and replayed in random order. Previous tests had an error variance of about 0.25.

   (a) What is the power when testing at the 0.01 level if the high cost tapes have an average one unit different from the low cost tapes?
   (b) How large should the sample size have been to have a 95% brand A versus brand B confidence interval of no wider than 2?

4. The following data are elasticity measurements on skin that has been exposed to varying intensities of light for a fixed time period. The light intensities are equally spaced but have been coded for convenience.

Light Intensity (Coded)	Elasticity Measurements						
1	0.54	1.98	0.65	0.52	1.92	1.48	0.97
2	1.76	1.24	1.82	1.47	1.39	1.25	1.29
3	2.05	2.18	1.94	2.50	1.98	2.17	1.83
4	7.92	4.88	9.23	6.51	6.77	4.25	3.72

(a) Evaluate whether there are significant differences in the elasticity measurements for the various light intensities.

(b) Check the ANOVA assumptions in your analysis. Suggest and implement any changes in the data that you find appropriate. Justify each step of your analysis. How do your conclusions change?

(c) Construct a confidence interval on the mean elasticity measurement for the highest light intensity. Is it significantly different from the other means of the other light intensities?

5. A clay tile company is interested in studying the effects of cooling temperature on strength. Since the company has five ovens which produce the tiles, four tiles were baked in each oven and then randomly assigned to one of the four cooling temperatures (degrees Celsius). The data are shown in the following:

Cooling Temp	Oven 1	2	3	4	5	Mean
5	3	10	7	4	3	5.40
10	3	8	12	2	4	5.80
15	9	13	15	3	10	10.00
20	7	12	9	8	13	9.80
Mean	5.50	10.75	10.75	4.25	7.50	7.75

(a) If $MS_E = 6.275$, compute the F-statistic to determine if there is a difference among the four cooling temperatures (use $\alpha = 0.05$). If significant, perform pairwise comparisons using Tukey's procedure.

(b) In this experiment, what is the power of detecting a difference of at least 6.0 units?

(c) Suppose the company believes there is a jump in the strength at 12.5 degrees Celsius but otherwise cooling temperature has no effect (i.e., step function ____——). Find a set of orthogonal contrasts that would allow you to test this. (HINT: What is the relationship among the means before and after 12.5 degrees?)

(d) Test these contrasts. State your conclusions.

6. Winter road treatments to clear snow and ice can lead to cracking in the pavement. An experiment was conducted comparing four treatments: sodium chloride, calcium chloride, a proprietary organic compound, and sand. Traffic level was used as a blocking factor and a randomized complete block experiment was conducted. One observation is missing, because the spreader in that district was not operating properly. The response is new cracks per mile of treated roadway.

	A	B	C	D
Block 1		32	27	63
Block 2	38	40	43	33
Block 3	40	63	14	26

One interest is in the following comparisons: chemical versus physical (A, B, C, versus D) inorganic versus organic (A, B, versus C), and sodium versus calcium (A versus B). Which of these comparisons seem large?

7. Shade trees in coffee plantations may increase or decrease the yield of coffee, depending on several environmental and ecological factors. Robusta coffee was planted at three locations in Ghana. Each location was divided into four plots, and threes were planted at densities of 185, 90, 70, and 0 trees per hectare. Data are the yields of coffee (kg of fresh berries per hectare) for the 1994-1995 cropping season (data from Amoah, Osei-Bonsu, and Oppong 1997):

Location	185	90	70	0
1	3107	2092	2329	2017
2	1531	2101	1519	1766
3	2167	2428	2160	1967

Analyze these data to determine the effect of tree density on coffee production.