Statistics 514: Problem Set No. 2

Due Thursday, September 13 (Session 8)

1. A cholesterol level above 200 mg/dl suggests a person is at a higher risk for heart disease. Suppose there is a test that measures a person's cholesterol level, but it is not perfect. Suppose the test's uncertainty is described by a normal distribution with the mean equal to the subject's true cholesterol level and a standard deviation of 7.5 mg/dl, what is the probability that

   (a) A subject with a true cholesterol level of 195 will be classified as being at higher risk for heart disease?

   (b) A subject with a true cholesterol level of 210 will not be classified at being at higher risk for heart disease?

   (c) A subject with a true cholesterol level of 210, using the average of five measurements, will not be classified as being at higher risk for heart disease?

   (d) A subject, with a true cholesterol level of 195, takes the test twice (assume independent) and each test classifies the subject incorrectly?

2. *Oehlert, page 29.* As part of a larger experiment, Dale (1992) looked at six samples of a wetland soil undergoing a simulated snow melt. Three were randomly selected for treatment with a neutral pH snow melt; the other three got a reduced pH snow melt. The observed response was the number of Copepoda removed from each microcosm during the first 14 days of snow melt. Test the hypothesis that the two treatments have equal means using randomization test methods. (Note: there are 20 different treatment allocations).

Reduced pH			Neutral pH		
256	159	149	54	123	248

3. Suppose that in the comparison of two treatments with r units for each treatment the observations are completely separated, for example that all the observations on T exceed all those on C. Show that the one-sided significance level under the randomization distribution is $(r!)^2/(2r)!$. Comment on the reasonableness or otherwise of the property that it does not depend on the numerical values and in particular on the distance apart of the two sets of observations.

4. The fuel economy of two lubricating oils for locomotive engines is to be investigated. Fuel economy is measured by determining the brake-specific fuel consumption (BSFC) after the oil has operated in the engine for 10 minutes. Each oil is to be tested five times. Suppose that the run order and the resulting data are as follows:

Run	BSFC	Run	BSFC
1	0.536	6	0.550
2	0.535	7	0.552
3	0.538	8	0.559
4	0.537	9	0.563
5	0.542	10	0.571

   Use graphical and numerical methods to discuss how conclusions about the effectiveness of the two oils might be compromised if the order that the oils were tested was

A, A, A, A, A, B, B, B, B, B.

Would the same difficulties occur if a randomization of the order of testing resulted in the following test sequence

A, B, A, A, B, B, A, B, B, A?

How would conclusions about the effectiveness of the two oils change, based on which test sequence was used?

5. Throughout this class, we make extensive use of the principle of least squares. In particular, we use the fact that the sample mean $\bar{Y}$ is the least-squares estimator of a population mean μ. This exercise explores this fact in additional detail from an empirical (as opposed to a mathematical) perspective. (Assume, for the sake of the problem, that the mean equals the median with probability 0.)

   (a) Suppose we have a sample of five scores: 43, 56, 47, 61, and 43. Calculate the sum of squared deviations from the mean of these five scores. Also, calculate the sum of squared deviations from the median for the five scores. Which is less? Will this always be true? Why or why not?

   (b) Suppose that we were to choose our estimator not to minimize the sum of squared errors, but instead to minimize the sum of the absolute values of the errors. Calculate the sum of absolute deviations from the mean and from the median. Which is less? Do you think this will always be true? Why or why not?

6. To determine whether waste discharged by a chemical plant is polluting the local river, the EPA plans to take water samples both upstream and downstream from the discharge site and measure the concentration level (ppm) of the suspected chemical pollutant. A two-sample t-test will be used to assess $H_0 : \mu_{\text{Down}} = \mu_{\text{Up}}$ vs $H_A : \mu_{\text{Down}} > \mu_{\text{Up}}$. You have been hired to determine the number of samples to take at each location and will use a modified version of tpower.sas to do these calculations. Since they are interested in a one-sided alternative, you need to eliminate the lower critical value (`rlow`) and compute the upper critical value using $1 - \alpha$ instead of $1 - \alpha/2$. This means the loop for sample size should be changed to

```
/* Sample size code for the > alternative */
do n = 2 to 11 by 1;
   df = 2*(n-1); nc = delta/(sigma*sqrt(2/n));
   rhigh = tinv(1 - alpha, df);
   p = 1 - probt(rhigh, df, nc); output;
end;
```

   (a) What sample size is needed to detect a change in true concentrations of 5 ppm 90% of the time when $\alpha = 0.05$ and $\sigma = 4.0$ (NOTE: You may need to change the range for n)?

   (b) Suppose instead they want to detect a change in true concentrations of 5 ppm 95% of the time. What value of n is needed?

   (c) Now consider that they want to detect a change in true concentrations of 5 ppm 95% of the time but set $\alpha = 0.10$. What value of n is needed?

(d) Using the results from the previous problems, what happens to n when all else is kept constant but the desired power is increased? What happens to n when all is kept constant but the α is increased?

7. An experiment is done where 5 treatments are randomly and equally assigned to 30 experimental units. Suppose that

$$\begin{aligned} \sum\sum(y_{i,j} - \bar{y}_{..})^2 &= 1230 \\ n\sum(\bar{y}_{i.} - \bar{y}_{..})^2 &= 650 \end{aligned}$$

Write out the ANOVA table and state whether there is at least one treatment effect different from zero at the 0.05 level.

8. The manufacturer of a breakfast cereal known as "A" is planning to test 6 new versions of its cereal. Call them "B", "C", "D", "E", "F", "G". This is to be done by a taste test, using expert cereal tasters. Design an experimental layout, using as few tasters as you can, subject to all of the following constraints:

- Each and every taster tastes exactly 3 cereals.
- No taster tastes any cereal twice.
- Each and every taster tastes cereal A.
- Each and every non-A cereal is tasted the same number of times.
- Each and every possible pair of non-A cereals is tasted together by the same number of tasters.
- Each and every non-A cereal (immediately) follows cereal A the same number of times.
- Each and every non-A cereal (immediately) precedes cereal A the same number of times.
- Each and every non-A cereal (immediately) precedes each other non-A cereal the same number of times.

Very strictly speaking, one could argue that using zero tasters satisfies the above. Restrict attention to designs with a positive number of tasters!