

**Statistics 514: Problem Set No. 1**  
Due Thursday, August 30 (Session 4)

Read: Chapters 1 and 2 of Montgomery.

Work (and turn in) the following Exercises on pages 54-59: 2-2, 2-9, 2-22, 2-25, 2-26, 2-30

1. Suppose I generate 10 random variables in the following manner. I first generate a variable  $X$  from a Normal distribution with mean 5 and variance 2. I then randomly sample 10 variables  $Y_1, Y_2, \dots, Y_{10}$  from a Normal with mean 0 and variance  $\frac{1}{2}$  and add  $X$  to each one. This creates 10 random variables

$$Z_i = X + Y_i \quad i = 1, 2, \dots, 10.$$

- (a) What is  $E(Z_i)$ ?
  - (b) What is  $\text{Var}(Z_i)$ ?
  - (c) What is  $E(\bar{Z})$ ?
  - (d) What is  $\text{Var}(\bar{Z})$ ?
2. *What's a degree of freedom worth?* Design books often take great pains to document the gain or loss of a degree of freedom. To see why people care, consider a  $\chi^2_{(d)}$  distribution, for various small  $d$ . Suppose that  $s^2$  is a random variable with distribution  $\sigma^2 \chi^2_{(d)}/d$ , where  $\sigma^2$  is a true variance of interest. For each  $d = 1, 2, 3, 4, 5, 10, 20, 100$ , form a central 95% confidence interval for  $\sigma^2$  assuming that  $s^2 = 1$ .

[Warning: Table III in Montgomery is labeled in a counterintuitive way, but it can be used if you're careful. Also, the four missing values denoted "0.00 +" are  $3.93 \times 10^{-5}$ ,  $1.57 \times 10^{-4}$ ,  $9.82 \times 10^{-4}$ , and  $3.93 \times 10^{-3}$ , to 3 significant figures.]

*In general, it takes a very large sample to learn a variance with any precision. But the effect can be drastic when the degrees of freedom are few.*

*It may be hard to know what a degree of freedom is worth, but it almost always costs one observation.*

*You can also see this effect by looking at the effects of the denominator degrees of freedom in an  $F$  table.*

3. *Graphing/Computing.* Construct a power curve for a two-sided  $t$ -test with  $\alpha = 0.01$ ,  $\sigma = 8$ , and  $n = 10$  (for a total of 20 observations) by calculating the power when  $\delta = 0, 2, 4, 6, 8, 10, 12, 14$ , and 16.