CONTROL CHARTS FOR SAMPLE PROPORTION

The p chart is a control chart which is used to monitor a process proportion. Examples: The proportion of parts from a milling machine which require rework. The proportion of orders filled within one day of receipt by LLBean.

The p chart is a “stand-alone” chart, i.e., it is not used in conjunction with another chart.

The p chart is an attribute chart, i.e., it is based on counts of items classified as defective, or late, etc., rather than measurements of a variable.

The p chart is used to plot the sample proportion, \( \hat{p} \), vs. time. Each sample represents a specific period of time such as 1) An 8 hour shift, 2) A working day, 3) A month.

The sample size is the number of items produced during the period of time, and the proportion plotted is the proportion of defective parts on the day shift, or the proportion of orders shipped on time.

The control limits for the p chart are based on the sample proportion being normally distributed. The centerline is the mean of the last 20-25 sample proportions, \( \hat{p} \). The upper and lower control limits are three standard deviations above and below the centerline.

The formulas for the limits are:

\[
\text{Calculate } \bar{p}, \text{ the mean of the last 20-25 values of } \hat{p} \\
UCL = \bar{p} + 3 \sqrt{\frac{\hat{p} (1-\hat{p})}{n}} \quad \text{but round down to 1.0 if formula gives a value > 1.0} \\
CL = \bar{p} \\
LCL = \bar{p} - 3 \sqrt{\frac{\hat{p} (1-\hat{p})}{n}} \quad \text{but round up to zero if formula gives a negative number}
\]
Evaluation of the p chart follows the same 3 criteria used for the Xbar-R or Xbar-S charts.

1. A single $\hat{p}$ value outside the control limits indicates the process proportion, $p$, has shifted and requires investigation.

2. A “run” of 9 consecutive $\hat{p}$ values on the same side of the centerline indicates the process proportion, $p$, has shifted and requires investigation.

3. A “trend” of 7 consecutive $\hat{p}$ values ascending or 7 consecutive $\hat{p}$ values descending indicates the process proportion, $p$, has shifted and requires investigation.

Some problems can occur which make the use of p charts difficult.

1. Recall that $\hat{p}$ is approximately normally distributed when:
   - $np \geq 15$, i.e., the number of bad parts or late shipments $\geq 15$
   - $n(1-p) \geq 15$, i.e., the number of good parts or on-time shipments $\geq 15$

   If the process does not produce 15 bad parts or 15 late shipments per time period, $\hat{p}$ will not be normally distributed and the time period must be increased.

2. The control limits depend on the number of items produced per time period, $n$, and if this is not constant, the control limits should be calculated for each period separately. But a chart with changing control limits is difficult to use.

   Sometimes an average value, $\bar{n}$ is used for the number of items produced per period.