10) If \( n = 1000 \) and \( p = .5 \)

What value of \( \hat{p} \) is equivalent to the 1st Quartile?

For 1st Q, \( z = -1.67 \)

\[
Z = \frac{\hat{p} - .5}{\sqrt{\frac{.5(1-.5)}{1000}}} = \frac{\hat{p} - .5}{.015811}
\]

\[
\hat{p} = .015811(z) + .5
\]

\[
\hat{p} = .015811(-1.67) + .5
\]

\[
\hat{p} = .4894
\]

11) Similarly, what value of \( \hat{p} \) is equivalent to the 90th percentile?

\[
Z = 1.28
\]

\[
\hat{p} = .5 + Z(.015811)
\]

\[
\hat{p} = .5 + 1.28(.015811)
\]

\[
\hat{p} = .5 + .0202
\]

\[
\hat{p} = .5202
\]
12) \[ \% \text{ CI for } p = \hat{p} \pm z^* \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} \]

We don't know \( p \) so we substitute \( \hat{p} \) 

\[ \% \text{ CI for } p = \hat{p} \pm z^* \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} \]

<table>
<thead>
<tr>
<th>% Conf</th>
<th>( Z^* )</th>
</tr>
</thead>
<tbody>
<tr>
<td>90</td>
<td>1.645</td>
</tr>
<tr>
<td>95</td>
<td>1.960</td>
</tr>
<tr>
<td>99</td>
<td>2.576</td>
</tr>
</tbody>
</table>

13) Example: \( n = 1000 \) \( \hat{p} = \frac{520}{1000} = .52 \)

95% CI for \( p = .52 \pm 1.960 \sqrt{\frac{.52(1-.52)}{1000}} \)

\[ = .52 \pm 1.960 (.015799) \]

\[ = .52 \pm .031 \quad .031 \text{ is margin of error} \]

95% CI = (.489, .551)
A 99% CI for $\rho =$

$\hat{\rho} \pm 2.576 (\sigma_{\hat{\rho}}) = 0.52 \pm 0.015799$

$= 0.52 \pm 0.01841$

$(0.479, 0.561)$

A 90% CI for $\rho =$

$\hat{\rho} \pm 1.645 (\sigma_{\hat{\rho}}) = 0.52 \pm 0.015799$

$= 0.52 \pm 0.026$

$(0.494, 0.546)$

Higher confidence makes $n$ larger
and makes CI wider

15)
Raise $n$ to 2000

$\hat{\rho} = 0.52$

95% CI for $\rho =$

$0.52 \pm 1.960 \sqrt{\frac{0.52(0.48)}{2000}}$

$= 0.52 \pm 0.01117$

$= 0.52 \pm 0.0219$

$(0.498, 0.542)$

Raising $n$ reduces $m$ and reduces CI width
16) Calculate \( n \) to control margin of error to a given maximum value.

\[
n = p^* (1 - p^*) \left( \frac{Z}{m} \right)^2
\]

- \( p^* \) is our estimate of \( p \)
- \( m \) is our maximum margin of error

17) Example: Calc \( n \) for a 95\% CI with \( m = .02 \)

Assume \( p^* = .5 \)

\[
n = .5 (1 - .5) \left( \frac{1.960}{.02} \right)^2
\]

\( n = 2401 \)

Always use \( p^* = .5 \) when \( p \) is unknown. This is the "safe" approach, i.e., it will give a value of \( n \) which will always keep \( m = .02 \) or less regardless of \( p^* \).
Hypothesis Tests

step 1 \( H_0: \ p = p_0 \)
\( H_A: \ p > p_0 \) \quad \text{choose the} \quad 1 \quad \text{which fits the situation}
\( p < p_0 \)
\( p \neq p_0 \)

step 2 Run survey, determine \( \hat{p} \)
No of "Yes" votes \( \geq 10 \)
and No of "No" votes \( \geq 10 \)

Test Statistic \( Z = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}} \)

step 3 Determine PValue

step 4 Reach decision on \( H_0 \).
IF PValue \( \leq \alpha \) REJECT \( H_0 \)
Example 1

$H_0: \, p = 0.5$

$H_A: \, p > 0.5$

$\alpha = 0.05$

We hope to show that a congressman has $>50\%$ support for re-election.

Data: $\hat{p} = \frac{520 \text{ "yes" votes}}{1000} = 0.52$

\[
Z = \frac{\hat{p} - p}{\sqrt{\frac{p(1-p)}{n}}} = \frac{0.52 - 0.5}{\sqrt{\frac{0.5(0.5)}{1000}}} = 0.02 = 1.26
\]

Since $P\text{value} > \alpha$ we cannot reject $H_0$.

Congressman may only have $50\%$ of electorate on his side.
20) Example 2

What if $\hat{p} = .53 \quad n = 1000$

$$Z = \frac{.53 - .5}{\sqrt{.5(1-.5)/1000}} = 1.90$$

If $\alpha = .05$ we can reject $H_0$ (Significant at .05)

If $\alpha = .01$ we cannot reject $H_0$ (Not significant at .05)