Stat 225 Solutions for the Final Exam Review Problems

For all problems, Z represents a standard normal variable and $\Phi(x)$=P($Z < x$).

1. Let X=student score; $X \sim$ Normal($\mu = 72$, $\sigma^2 = 8^2$).

   (a) P($X < 50$)=P($Z < \frac{50-72}{8}$)=$\Phi(-2.75)$=0.0030.
   Hence, we would expect 0.3 students in 100 to score this low. Remember that our expected value nned not be a whole number.

   (b) Let c be the cutoff score. P($X > c$)=0.75 $\rightarrow$ P($X < c$)=0.25 $\rightarrow$ P($Z < \frac{c-72}{8}$)=0.25 $\rightarrow$ $\frac{c-50}{8}$=$\Phi^{-1}(0.25)$=-0.674.
   Therefore, $c = 72 - 8 * 0.674$=66.6.

2. We have $X \sim$ Bin($n = 360$, $p = \frac{1}{36}$). Since $np \geq 10$ and $n(1-p) \geq 10$, we can approximate X by $X^* \sim$ Normal ($\mu = 10$, $\sigma^2 = 10 * \frac{35}{36}$=9.722.)

   (a) P($8 \leq X \leq 12$) = P($7.5 < X^* < 12.5$) = P($\frac{7.5-10}{\sqrt{9.722}} < Z < \frac{12.5-10}{\sqrt{9.722}}$)= $\Phi(0.801)$ - $\Phi(-0.801)$ = 0.789 - 0.211 = 0.577.

   (b) P($8 < X < 12$) = P($8.5 < X^* < 11.5$) = P($\frac{8.5-10}{\sqrt{9.722}} < Z < \frac{11.5-10}{\sqrt{9.722}}$)= $\Phi(0.481)$ - $\Phi(-0.481)$ = 0.685 - 0.315 = 0.370.

3. Since P(A) + P(B) = P($A \cap B$) + P($A \cup B$) = 1 and P(A)> P(B), then we must have P(A)> 0.50 and P(B) < 0.50. Furthermore, P($A \cup B$) $\geq$ P(A) and P($A \cap B$) $\leq$ P(B). Therefore, $0.50 < P(\text{A}) \leq 0.75$, $0.25 \leq P(\text{B}) < 0.50$, with the additional restriction that P(B) = 1 - P(A).

4. (a) $\binom{4}{3}(.5)^3 * (.5)$=0.25.

   (b) $(\frac{7}{8})^4$=0.586.

   (c) $(\frac{1}{8})^4$=0.00024

5. (a) By the Basic Counting Rule, he has 3*6*2*3 = 108 possible choices.

   (b) Each of the 108 choices in a) can be ordered (i.e. permuted) 4!, or 24, distinct ways. Therefore, he has 104*24 or 2,592 ways.

   (c) He has $\binom{14}{4}$ or 1001 different ways to select four countries.

   (d) There are 14^4, or 38,416, equally likely ways of selecting four countries with replacement. In 11^4, or 14,461, of these outcomes there are no South American countries selected. The probability of not selecting a South American country is $\frac{14461}{38416}$, or 0.3811.

6. If $Y \sim$ Geometric(p), then E(Y)= $\frac{1}{p}$ and Var(Y) = $\frac{1-p}{p^2}$. If Var(Y) = 144, then $144p^2 + p - 1 = 0$ and by the quadratic equation, we have $p = 0.0799$. Hence, the pmf is $p_Y(y)$=$(0.9201)^{y-1} * (0.0799)$ for $y = 1, 2, 3...$.

7. (a)

x	$p(x)$
40	0.004
30	0.0362
20	0.0498
19	0.109
9	0.326
-2	0.475

(b) $E(X) = 6.31$ and $Var(X) = 87.44$.

(c) $E(W) = 63.1$ and $Var(W) = 874.4$.

8. (a) Poisson($\lambda = 450$)

   (b) Binomial($n = 100, p = 0.95$)

   (c) Binomial($n = 19, p = 0.565$) - consider only consecutive voters

   (d) Exponential($\lambda = 67.5$) (time in hours)

   (e) Normal($\mu = 4750, \sigma^2 = 237.5$)

   (f) Uniform[9:00, 12:00]

   (g) Geometric($p = 0.0275$)

9. The probability of rolling an even number is 0.5. By the CLT, the proportion of even rolls out of n trials will be distributed Normal with $\mu = 0.50$ and $\sigma^2 = \frac{0.25}{n}$. To get a 90% probability of being between 48% and 52%, we must make 52% our 95th percentile. (This is because we are centered around our μ.)
   $0.50 + \Phi^{-1}(0.95)*\frac{0.5}{\sqrt{n}} = 0.52$.
   $1.645*\frac{0.5}{\sqrt{n}} = 0.02$.
   $\frac{0.5}{\sqrt{n}} = \frac{0.02}{1.645} = 0.012158$
   $n = (\frac{0.5}{0.012158})^2$=1691.26.
   So you would need to simulate at least 1692 rolls.

10. $X \sim$ Exponential ($\lambda = \frac{1}{120}$).

   (a) 1 - $e^{\frac{-120}{120}})^6$ = 1 - 0.0637 = 0.9363.

   (b) P($X > 240$) = $e^{\frac{-240}{120}}$ = 0.135.
   Let W be the number of bulbs until third bulb that lasts 240 hours. $W \sim$ Negative Binomial ($r = 3$, $p = 0.135$). E(W) = $\frac{3}{0.135}$ = 22.22.

   (c) 1 - $(1 - 0.135)^6$ = 0.419.

11. (a) $k = \frac{10}{784}$

   (b) $$F(x) = \begin{cases} 0 & x \leq 2 \\ \frac{10}{784}[\frac{1}{5}x^5 - \frac{1}{2}x^4 + \frac{8}{5}] & 2 < x \leq 4 \\ 1 & x > 4 \end{cases}$$

   (c) E(X) = 3.5102

   (d) $P(2 < X < 3) = 0.1237$

   (e) $P(X > 3.5) = \frac{0.3483}{1-0.0837}$=0.5968

12. (a) The PDF is below.
   $$f(x) = \begin{cases} \frac{3x^2}{8} & 0 < x \leq 2 \\ 0 & \text{otherwise} \end{cases}$$

   (b) E(X) = $\int_0^2 \frac{3}{8}x^3 dx = \frac{48}{32}$ = 1.500.

   E(X^2) = $\int_0^2 \frac{3}{8}x^4 dx = \frac{96}{40}$ = 2.400.

   Var(X^2) = E(X^2) - [E(X)]^2 = 2.400 - 1.500^2 = 0.15.

(c) $P(0 < X < 0.5) = \int_0^{0.5} \frac{3}{8}x^2 dx = \frac{1}{64} = 0.0156.$

(d) $P(0 < X < 0.6) = \int_0^{0.6} \frac{3}{8}x^2 dx = 0.027.$

P(both are greater than 0.6)=(1 - 0.027)^2 $= 0.9467.$

13. (a) Let X = number of balls sunk. Then $X \sim$ Binomial(3,0.2) and $P(X \geq 1) = 0.488.$

(b) Let Y = number of contestants until the third one wins. Then $Y \sim$ Negative Binomial(3, 0.488) and $E(Y) = 3 \cdot \frac{1}{0.488} = 6.148$

(c) Let X = number of contestants among the 300 who win. Then $X \sim$ Binomial(300, 0.488) $\overset{approx.}{\sim}$ Normal(146.4, 74.9568). Hence

$$P(X \geq 150) = P\left(Z > \frac{149.5 - 146.4}{8.658}\right) = 1 - P(Z \leq 0.358) = 1 - 0.64 = 0.36$$

14. (a) $\frac{1}{3} * 0.50 + \frac{1}{3} * 1 + 0 = 0.50.$

(b) $\frac{\frac{1}{3}*0.50}{0.50} = \frac{1}{3}.$

(c) Our answers to (a) and (b) will not change because the conditional probability of drawing white from Urn III will still be 1.

15. (a) No, X and Y are not independent. As one example, $p(0,10) = 0 \neq p_X(1)p_Y(10) = 0.30 * 0.20 = 0.06.$

(b) E(X)=1, E(Y)=18.

(c) Var(X)=0.60, Var(Y)=16 .

(d) Cov(X,Y)= E(XY) - E(X)E(Y) = 0 (X and Y are still not independent.)

(e) Var($Y - 2X$) = Var(Y) + 4*Var(X) -2*2*Cov(X,Y) = 16 + 4*0.60 -2*0 = 18.40.

16. 0.800.

17. (a) FALSE

(b) TRUE

(c) TRUE

(d) TRUE

(e) FALSE

(f) TRUE

(g) FALSE

(h) TRUE

(i) FALSE

(j) TRUE

(k) TRUE

(l) FALSE

(m) FALSE

(n) FALSE

(o) FALSE

(p) TRUE

(q) FALSE

(r) TRUE

18. Let T be the total weight of all 25 guests. By the CLT, the sum of all the guests weights is distributed Normal with $\mu = 25 * 160 = 4000$ and $\sigma^2 = 50^2 * 25 = 250^2$.) Therefore P($T > 4500$) =P($Z > \frac{45000-4000}{250}$ = 1 - $\Phi(2)$ = 1 - 0.987 = 0.023.

19. We have X distributed Binomial (n=2000, $p = \frac{18}{38} = 0.4736$). Since we expect to have at least 10 successes and 10 failures, we can approximate X with X^*, where X^* is distributed Normal ($\mu = 2000{*}0.4736 = 947.3$, $\sigma^2 = 2000{*}0.4736{*}0.5264 = 498.7$). Hence the probability we have between 950 and 1050 wins is computed as follows: $P(950 \le X \le 1050) = P(949.5 < X^* < 1050.5) = P(\frac{949.5-947.3}{\sqrt{498.7}} < Z < \frac{1050.5-947.3}{\sqrt{498.7}})$ $= \Phi(4.5899)$ - $\Phi(0.0985)$ = 1 - 0.5392 = 0.4608.

20. (a) 95%C.I. for μ is $\bar{X} \pm 1.96{*}\frac{\sigma}{\sqrt{n}}$.
    Therefore, we have a lower limit of 12.04 - $1.96{*}\frac{0.08}{\sqrt{64}} \approx 12.02$.
    We have an upper limit of 12.04 + $1.96{*}\frac{0.08}{\sqrt{64}} \approx 12.06$.
    Our 95% C.I. for μ is between 12.02 and 12.06 oz.

    (b) Since 12.00 is not in our interval, it is not reasonable (in a statistical sense) to call them 12 oz. bottles. This difference is not practically important, however.

21. 99%C.I. for the true mean study time is $\bar{X} \pm 2.58{*}\frac{\sigma}{\sqrt{n}}$
    $142 \pm 2.58{*}\frac{16}{\sqrt{72}} = (137.14, 146.86)$