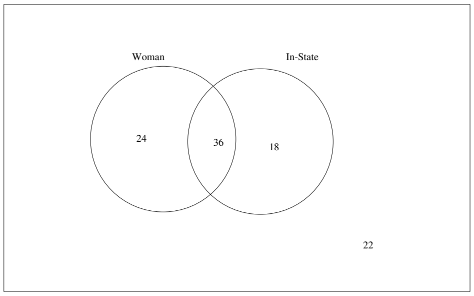
1. Upper left: $(A \cap B^c) \cup (A^c \cap B)$.
   Upper right: $B \cap (A \cup C)$.
   Lower left: $B \cap A^c \cap C^c$.
   Lower right: $B \cap (A \cup C)$.

2. Suppose we select one student at random from the class. Let W be the event the student is a woman and let S be the event that the student is from in-state. We then have $P(S|W) = 0.60$ and $P(S|W^c) = 0.52$.

   (a)



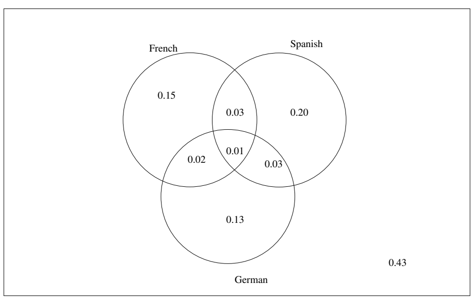
Venn Diagram for Review Problem 2

   (b) $P(S|W) = 0.60$, which is given explicitly.

   (c) $P(W|S^c) = \frac{P(W \cap S^c)}{P(S^c)} = \frac{\frac{24}{100}}{\frac{46}{100}} = \frac{24}{46} = 0.5217$.

   (d) Since $P(W^c \cap S) = \frac{18}{100} = 0.18$ does not equal $P(W^c) * P(S) = \frac{40}{100} * \frac{54}{100} = 0.216$, the events are not independent.

3. (a)



Venn Diagram for Review Problem 3

   (b) 0.43.

   (c) $\frac{4\%}{27\%} = 0.1481$.

   (d) $\frac{1\%}{4\%} = 0.25$.

4. Suppose $P(E) = 0.35$, $P(F) = 0.70$, and $P(E \cap F) = 0.23$. Compute the following probabilities (a Venn diagram may help):

   (a) $P(E|F) = \frac{P(E \cap F)}{P(F)} = \frac{0.23}{0.70} = 0.3286$.

   (b) $P(F|E) = \frac{P(F \cap E)}{P(E)} = \frac{0.23}{0.35} = 0.6571$.

(c) $P(F|E^c) = \frac{P(F \cap E^c)}{P(E^c)} = \frac{P(F) - P(F \cap E)}{P(E^c)} = \frac{0.70 - 0.23}{1 - 0.35} = 0.7231$.

(d) $P(E \cap F|$ at least one event occurs $) = \frac{P(E \cap F)}{1 - P(E^c \cap F^c)} = \frac{0.23}{1 - 0.18} = 0.2804$.

5. (a) No, may only be equal to $P(A \cap B)$ by chance.
   (b) No, this is equal to $P(A \cup B)$. This is only equal to $P(A \cap B)$ if $P(A \cap B^c) = P(A^c \cap B) = 0$.
   (c) Yes, equal by definition.
   (d) No, this is equal to $P(A \cup B)$.
   (e) No, never equal to $P(A \cap B)$.
   (f) No, only equal to $P(A \cap B)$ if A and B are independent events.
   (g) No, may only be equal to $P(A \cap B)$ by chance.
   (h) Yes, equal by the General Multiplication rule.
   (i) No, never equal to $P(A \cap B)$. This is , in fact, equal to - $P(A \cap B)$.
   (j) Yes, equal because $(A^c \cup B^c)$ is the complement of $(A \cap B)$ from DeMorgan's Second Law.

6. There are three possibilities for each of the seven balls, so by the Basic Counting Rule, there are 3^7, or 2187 total outcomes. Since the tosses are random, each outcome is equally likely. We need to determine how many of these outcomes will have one basket with exactly five balls. We start with the probability that there are exactly five balls in the first basket.
   P(first basket contains exactly five balls) = $\frac{\text{Number of favorable outcomes}}{\text{Total number of outcomes}}$

   Number of favorable outcomes = Number of ways of getting exactly five balls in first basket*number of ways of getting the other two balls in other baskets = $\binom{7}{5} * 2^2 = 21 * 4 = 84$.
   P(first basket contains exactly five balls) = $\frac{84}{2187} = 0.0384$.

   Similarly, we have P(second basket contains exactly five balls) = P(third basket contains exactly five balls) = 0.0384. Since there are only seven balls, at most one basket can contain five of them.
   P(one basket contains exactly five balls) = $3 * 0.0384 = 0.1152$.

7. (a) P(exactly one R) = $\frac{\binom{20}{1} * \binom{33}{4}}{\binom{53}{5}} = \frac{20 * 40{,}920}{2{,}869{,}685} = 0.2852$.
   (b) P(two or more D) = 1 - P(exactly one D) - P(exactly 0 D) = $1 - \frac{\binom{33}{1} * \binom{20}{4}}{\binom{53}{5}} - \frac{\binom{20}{5} * \binom{33}{0}}{\binom{53}{5}} = 1 - \frac{159{,}885 + 15{,}504}{2{,}869{,}685} = 1 - 0.06112 = 0.9389$.

8. A sub will contain one meat on one bread with any number of additions. Therefore, we have
   Number of different subs = Number of meats * Number of breads*(Addition 1- yes or no)*(Addition 2 - yes or no) ... (Addition 12 - yes or no) =
   $6 * 4 * 2^{12} = 98,304$

9. The first three prizes are ordered, so we need to use permutations.
Number of distinct ways to award Best, Second, and Third = ${}_{18}P_3$=18 $*$ 17 $*$ 16=4, 896.
After the three top prizes are awarded, the Honorable Mention prizes are unordered. We need to use combinations.
Number of distinct ways to award two HM = ${}_{15}C_2 = \frac{15!}{13!*2!} = \frac{15*14}{2!}$=105.
Hence the total number of ways the judges can award all prizes is 4, 896 $*$ 105, or 514, 080.

10. There are 4 E's, 2 N's, 2 S's, and 1 T in the 9 letters of TENNESSEE. Hence there are $\frac{9!}{4!*2!*2!*1!}$=3780 distinguishable permutations. We answer the following questions by determining how many of these permutations will be 'favorable.'

    (a) There are six places the four E's can "start". From each of these six places, the remaining 5 letters can be permuted in $\frac{5!}{2!*2!*1!}$ or 30, distinguishable ways. Hence, there are 6 $*$ 30 or 180 ways of having all the E's together, and the probability of this happening is $\frac{180}{3,780} = \frac{1}{21}$.

    (b) If the S's and the T are all placed, there are $\frac{6!}{4!*2!}$, or 15, different ways for the other letters to be placed. Since this number will be the same for all permutations of S_1, S_2, and T, it can be divided out. The probability that T comes last from these three letters is just $\frac{1}{3}$.

    (c) By the same reasoning as (b), this is the probability that T is not last among the three letters. This probability that T Is last is $\frac{1}{3}$, so the probability T is not last is $1 - \frac{1}{3}$, or $\frac{2}{3}$.

    (d) If the N's are in the first and last place, there are $\frac{7!}{4!*2!*1!}$, or 105 distinct ways of placing the remaining letters. The probability of this happening is therefore $\frac{105}{3780}$, or $\frac{1}{36}$.

11. Each of the first four digits has five possibilities. The fifth and six digits have 99 possibilites, since they cannot be 00 together (this does not mean that neither can be 0.) Each of the final four digits has nine possibilities. Hence by the Basic Counting Rule, we get the following.
Number of DL #'s = $5^4 * 99 * 9^4 = 405,961,875$.

12. Let F be the event you chose the first crate, and let A be the event you chose an apple. From the randomization scheme, we know $P(F)$=0.75 and $P(F^c)$=0.25. We can also compute the conditional probabilities $P(A|F)$, $P(A^c|F)$, $P(A|F^c)$, and $P(A^c|F^c)$. For example, $A|F^c$ is the event that we draw an apple from the second crate. We can then calculate $P(A|F)$=0.40, $P(A^c|F)$=0.60, $P(A|F^c)=\frac{30}{70}$=0.429, and $P(A^c|F^c)=\frac{40}{70}$=0.571.

    (a) $P(A^c)=P(A^c|F) * P(F) + P(A^c|F^c) * P(F^c) = 0.60 * 0.75 + 0.571 * 0.25 = 0.593$.

    (b) $P(F|A^c)=\frac{P(A^c|F)*P(F)}{P(A^c|F)*P(F)+P(A^c|F^c)*P(F^c)} = \frac{0.60*0.75}{0.60*0.75+0.571*0.25} = \frac{0.450}{0.593}$=0.759.

    (c) $P(F|A)=\frac{P(A|F)*P(F)}{P(A|F)*P(F)+P(A|F^c)*P(F^c)} = \frac{0.40*0.75}{0.40*0.75+0.429*0.25} = \frac{0.30}{0.407}$=0.737.

(d) The events in (b) and (c) are not complements of each other. They are not conditioned on the same event.

13. We choose one child at random. Let H be the event that the child is a Harry Potter fan, and let F be the event that that child is female. We find that $P(H)$=0.65, and $P(H \cap F)$=0.29. Hence we use the formula for conditional probability to get the following:

$$P(F|H) = \frac{P(F \cap H)}{P(H)} = \frac{0.29}{0.65} = 0.4462$$

.

14. Let W_1 be the event that Team A wins the first game, and W_2 be the event that Team A wins the second game.

    (a) $P(W_1)$=0.72, $P(W_2|W_1)$=0.44, and $P(W_2|W_1^c)$=0.87.

    (b) $P(W_1 \cup W_2) = 1 - P(W_1^c \cap W_2^c) = 1 - P(W_2^c|W_1^c) * P(W_1^c) = 1 - (1 - 0.72) * (1 - 0.87) = 1 - 0.28 * 0.13 = 0.9636$.

    (c) $P(W_1 \cap W_2) = P(W_2|W_1) * P(W_1) = 0.72 * 0.44 = 0.3168$.

15. There are sixty-four possible outcomes. If the die are fair, each is equally likely. Let A_1 be the event that the first die is a 1 or 2, and let A_2 be the event that the second die is a 1 or a 2. We then know $P(A_1) = P(A_2)$=0.25. It is reasonable to assume the events are independent.

    (a) Since A_1 and A_2 are independent, we have $P(A_1^c \cap A_2^c) = P(A_1^c) * P(A_2^c) = (1 - 0.25) * (1 - 0.25) = 0.5625$. This is also equal to $\frac{36}{64}$, since there are $6 * 6$ different outcomes without a 1 or 2.

    (b) If the two faces are different, then we remove eight of the sixty four outcomes. There are then $6 * 5$, or 30 favorable outcomes, so the probability is $\frac{30}{56}$, or 0.5357.

16. Let M_k be the event that we choose matchbox k. Since the selection is random, we have $P(M_1)$=$P(M_2)$=$P(M_3)$=$\frac{1}{3}$. Let D be the event that we choose a dry match. We can compute the following conditional probabilities: $P(D|M_1)$=0.90, $P(D|M_2)$=0.85, and $P(D|M_3)$=0.

    (a) $P(D) = P(D|M_1) * P(M_1) + P(D|M_2) * P(M_2) + P(D|M_3) * P(M_3) = 0.90 * \frac{1}{3} + 0.85 * \frac{1}{3} + 0 = 0.5833$

    (b) $P(M_1|D) = \frac{P(D|M_1)*P(M_1)}{P(D|M_1)*P(M_1)+P(D|M_2)*P(M_2)+P(D|M_3)*P(M_3)} = \frac{0.30}{0.5833} = 0.5143$. Since $P(M_3|D) = 0$, we know $P(M_2|D) = 1 - 0.5143 = 0.4857$. This is because M_1, M_2, and M_3 form a partition.

    (c) Let X be the event that you draw one dry and one wet match. We need to find $P(X|M_1)$ and $P(X|M_2)$. Obviously, $P(X|M_3)$ is still 0.
    We find $P(X|M_1)$=$\frac{9*1}{_{10}C_2} = \frac{9}{45} = 0.20$.
    We find $P(X|M_2)$=$\frac{17*3}{_{20}C_2} = \frac{51}{190} = 0.2684$
    Therefore $P(M_1|X) = \frac{P(X|M_1)*P(M_1)}{P(X|M_1)*P(M_1)+P(X|M_2)*P(M_2)} = \frac{0.20}{0.4684}$=0.4269.

17. There are $\binom{9}{2}$, or 36 ways of selecting two tiles from the nine remaining. Order is not important. To answer each of the questions below, we need to determine how many of these draws will give the desired result.

    (a) $\frac{3*6}{36}$=0.5.

    (b) $\frac{3*2}{36}$=0.167.

    (c) $\frac{\binom{3}{2}}{36} = 0.0833$.

18. Let A be the event that the hotel guest is American. Then we have $P(A)$=0.30 and $P(A^c)$=0.70. Let V be the event that a randomly chosen letter from the written word (rigor or rigour) is a vowel. Therefore $P(V|A)=\frac{2}{5}$=0.40, and $P(V|A^c)=\frac{3}{6}$=0.50. We find $P(A|V)$ by Bayes Rule.
    $P(A|V)=\frac{P(V|A)*P(A)}{P(V|A)*P(A)+P(V|A^c)*P(A^c)}=\frac{0.40*0.30}{0.40*0.30+0.50*0.70}=\frac{0.12}{0.47}$=0.2553.

19. There are at least two ways to solve this problem. Note that $K is shorthand for thousands of dollars.
    Method 1: Suppose the boss gives the first employee $1K. Then the second employee can get anywhere between $1K and $10K, with the third employee getting whatever is left. So we have 10 possibilities. If the boss gives the first employee $2K, there are only 9 possibilities for the other two employee. Hence, there are a total of $10 + 9 + 8 + 7 + 6 + 5 + 4 + 3 + 2 + 1$, of 55 total possibilities.

    Method 2: The boss can select two 'stopping points'. (Picture a stack of thousand-dollar bills, even though they have been out of circulation for some time.) He can stop at any two different places between 1 and 11 for the first two employees and the third one gets whatever is left. Hence the number of possibilities is ${}_{11}C_2$, or 55.

20. Let B be the event you select a boy and let S event that the selected child prefers strawberry. We have $P(B) = \frac{16}{32}$, or 0.50, and $P(S)=\frac{14}{32}$, or 0.4375. If gender and ice-cream preference were independent, then $P(B \cap S)$ would equal $\frac{16}{32} * \frac{14}{32}$, or 0.2188, but we have 6 boys who prefer strawberry in the class. Therefore, $P(B \cap S) = \frac{6}{32} = 0.1875$, so gender and ice-cream preference are not independent.