1. The final scores (exams, quizzes and HWs combined) of students in a large chemistry class are approximately normally distributed with a mean of 72 points and a standard deviation of 8.

   (a) How many students in a section of 100 do you expect to end up with a score below 50?

   (b) Where is the highest the C cutoff could be made and still have at least 75% of the students get a C or better?

2. A set of fair die is tossed repeatedly 360 times. Let X be the number of times "boxcars" (double-sixes) are tossed. By using the Normal approximation to the Binomial, compute

   (a) $P(8 \leq X \leq 12)$

   (b) $P(8 < X < 12)$

3. Suppose $\mathrm{P}(A \cap B) = 0.25$ and $\mathrm{P}(A \cup B) = 0.75$. If $\mathrm{P}(A) > \mathrm{P}(B)$, then what are the possible values of $\mathrm{P}(A)$ and $\mathrm{P}(B)$?

4. A fair eight-sided die (numbered 1-8) is tossed four times. What is the probability of each of the following events?

   (a) There are exactly three even numbers.

   (b) There are no eights.

   (c) The die comes up 3, 4, 2, 6 in that order.

5. Joe is planning his future vacations. There are three countries he'd like to visit in South America, six in Europe, two in Africa, and three in Asia. He wants to visit one country on each continent over the next four years.

   (a) How many different ways can he do this?

   (b) If he must decide in advance which year he visits each country, how many ways can he do this?

   (c) How many ways can he choose four countries, regardless of continent?

   (d) Suppose he selects four countries randomly, with replacement. What is the probability that he does not select a South American country?

6. A geometric random variable has a variance of 144. Find its PMF.

7. A game consists of drawing two cards from a standard deck of 52 cards. For every face card you win \$10 and for an ace you win \$20. For any other card you lose \$1. Let X be the amount of money you win.

   (a) Find the PMF table for X.

   (b) Compute the expected value and variance of your winnings.

   (c) If you play this game repeatedly 10 times (the cards are put back after each game), what are the expected value and variance of your **total** winnings?

8. On election day, voters arrive at a polling place according to a Poisson process with an average rate of 150 every hour. Suppose that in this particular precinct, 55% of the registered voters vote republican and 45% vote democratic (ignore all other parties). Suppose that the new electronic voting machines correctly count a vote with probability 0.95.

   For each of the following random variables decide which distribution best describes the situation and find the values of all relevant parameters.

   (a) Let X be the number of votes cast during the first three hours after the polling place opens.

   (b) Let X be the number of correctly counted votes among the first 100 votes cast.

   (c) Let X be the number of times that two voters cast their votes within 20 second of each other among the first 20 voters.

   (d) Let X be the time between the next two *democratic* votes.

   (e) If all 5000 registered voters in this precinct cast their votes, let X be the number of votes correctly counted by the machines.

   (f) The local candidate for state representative casts his vote at some time between 9:00 a.m. and 12:00 p.m. Let X be the exact time at which he casts his vote.

   (g) Let X be the number of votes cast until somebody votes republican and the vote is not correctly counted.

9. Suppose you are simulating a large number of die rolls. How many simulations would you have to run to be 90% sure your proportion of even rolls is between 48% and 52%?

10. The lifetimes of a certain make of light–bulbs are a random variable with an exponential distribution with mean 120 days.

    (a) If a bulb burns out, it is immediately replaced by another. If I buy a six-pack of these bulbs, what is the probability that at least one of them will last 120 days?

    (b) How many bulbs can I expect to replace before finding the third one that lasts longer than 240 days?

    (c) What is the probability that at least one of the bulbs in the six-pack will last 240 days?

11. A continuous random variable X has PDF

$$f(x) = \begin{cases} kx^3(x-2) & 2 \leq x \leq 4 \\ 0 & \text{otherwise} \end{cases}$$

    (a) Find the value of k that makes this a valid PDF.

    (b) Compute the CDF.

    (c) Find the mean of this distribution.

    (d) Compute the probability that X is between 2 and 3.

    (e) Compute the probability that X is greater than 3.5

12. A continuous random variable X has the following CDF

$$F(x) = \begin{cases} 0 & \leq 0 \\ x^3/8 & 0 < x \leq 2 \\ 1 & x > 2 \end{cases}$$

(a) Find the PDF of X.

(b) Find the mean and standard deviation of this distribution.

(c) Compute the probability that X is between 0 and 0.5.

(d) Suppose we take two independent observations from this distribution. What is the probability that both are greater than 0.60?

13. On a game show contestants play a game by putting minigolf balls from a five yard distance. Every contestant gets three chances to make a put. Assume that every contestant has an equal chance of 0.2 of sinking a ball. The game is won, if at least one of the three balls can be sunk.

(a) What is the probability that a contestant wins this game? Which distribution are you using and what are the parameters?

(b) How many contestants do you expect to see on average until the third one wins? Which distribution are you using and what are the parameters?

(c) If 300 contestants line up to play this game, what is the approximate probability that at least 150 of them win?

14. Urn I contains five black and five white balls. Urn II contains one black ball. Urn III contains two white balls. One Urn is chosen at random and one ball is taken at random from the chosen urn.

(a) What is the probability that the selected ball is white?

(b) Given that the selected ball was black, what is the probability that it came from Urn I?

(c) Suppose we take one white ball away from Urn III and run the sample again. Do our answers in (a) and (b) change? Why or why not?

15. Let X and Y be discrete random variables with joint probability distribution function

$p_{X,Y}(x,y)$	0	1	2
10	0	0.20	0
20	0.30	0.20	0.30

(a) Are X and Y independent?

(b) Find $E(X)$ and $E(Y)$.

(c) Find $Var(X)$ and $Var(Y)$.

(d) Find $Cov(X,Y)$.

(e) Find $Var(Y - 2X)$.

16. Suppose X has a Uniform(0,1) distribution. Find $EX^{\frac{1}{4}}$.

17. A continuous random variable X has PDF $f(x)$ and CDF $F(x)$. Decide for each of the following statements if they are TRUE or FALSE.

   (a) $F(x)$ can have the value 2.5.
   (b) $f(x)$ can have the value 2.5.
   (c) $F(x)$ can be 1 for two values of x.
   (d) $f(x)$ can be 1 for two values of x.
   (e) The graph of $F(x)$ can have ups and downs.
   (f) The graph of $f(x)$ can have ups and downs.
   (g) The graph of $F(x)$ can have jumps.
   (h) The graph of $f(x)$ can have jumps.
   (i) The integral of $F(x)$ is $f(x)$.
   (j) The integral of $f(x)$ is $F(x)$.
   (k) The derivative of $F(x)$ is $f(x)$.
   (l) The derivative of $f(x)$ is $F(x)$.
   (m) $F(x)$ can be negative.
   (n) $f(x)$ can be negative.
   (o) The area under the curve $F(x)$ is 1.
   (p) The area under the curve $f(x)$ is 1.
   (q) $f(3)$ is the same as $P(X = 3)$.
   (r) $F(3)$ is the same as $P(X < 3)$.

18. A new elevator in a large hotel is designed to carry about 25 people, with a total weight of up to 4500 lbs. The average weight of guests at this hotel is 160 lbs. with a standard deviation of 50 lbs. Suppose 25 of the hotel's guests get into the elevator. Assuming the weights of these guests are independent random variables, what is the probability of overloading the elevator?

19. The probability of a Red bet winning in Roulette is $\frac{18}{38}$, or 0.473. Let X be the number of wins in 2000 Red bets. Find the approximate probability that you win between 950 and 1050 bets.

20. The amount of water in a '12-oz.' water bottle follows a normal distribution with unknown mean μ oz and standard deviation σ=0.08. We take a random sample of 64 water bottles and get a sample mean of 12.04 oz.

   (a) What is the distribution of the sample mean? What are you using to find this distribution?
   (b) Find a 95% confidence interval for the average amount of water in a bottle.
   (c) Based on your (a, is it reasonable to call them '12 oz. bottles'?

21. For a specific department, the amount of time a graduate student studies for a qualifying exam has a standard deviation of 16 hours. A sample of 72 students has an average of 142 hours studying. Find the 99%C.I. for the true average amount of a graduate student will study for this exam.