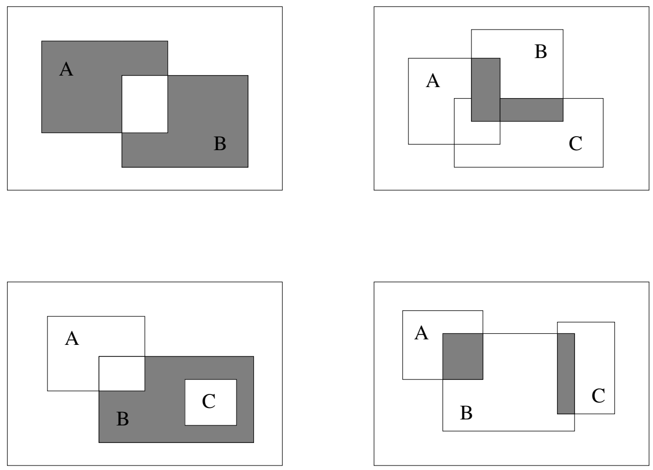
1. For each of the following four figures, find the expression in set notation that describes the shaded set.



2. A large psychology class has 60 women and 40 men. Sixty percent of the women come from in-state, as do forty-five percent of the men.

   (a) Draw a two-way Venn Diagram describing the situation and fill in all four regions.

   (b) What is the probability that a randomly chosen woman is from in-state?

   (c) What is the probability that a randomly chosen out-of-state student is a woman?

   (d) Are the events "Student is a man" and "Student is from in-state" independent? Why or why not?

3. Suppose that a random sample of a certain population of college students showed that 27% of them studied Spanish, 21% have studied French, and 19% have studied German. Furthermore, it showed that 4% studied French and Spanish, 3% studied French and German, and 4% studied German and Spanish. There was 1% of the study that studied all three. Suppose we choose a college student from this population at random.

   (a) Draw a three-way Venn Diagram and fill in as many spaces as possible.

   (b) What is the probability that the student studied none of the three?

   (c) Given that the student had studied Spanish, what is the probability that he or she also studied French?

   (d) Given that the student had studied Spanish and French, what is the probability that he or she also studied German?

4. Suppose $P(E) = 0.35, P(F) = 0.70,andP(E \cap F) = 0.23$. Compute the following probabilities (a Venn diagram may help):

   (a) $P(E|F)$.

(b) $P(F|E)$.

(c) $P(F|E^C)$.

(d) $P(E \cap F|$ at least one event occurs)

5. Let A and B denote events. Assume $P(A) > 0$ and $P(B) > 0$, but not assume $P(A \cap B) > 0$. For each of the following expressions, decide if it is ALWAYS equal to $P(A \cap B)$.

   (a) $P($ exactly one of A, B occurs$)$

   (b) $P($ at least one of A, B occurs$)$

   (c) $P($ both A and B occur$)$

   (d) $1 - P(A^C \cap B^C)$

   (e) $P(A) + P(B)$

   (f) $P(A) * P(B)$

   (g) $P(A) * P(A|B)$

   (h) $P(A) * P(B|A)$

   (i) $P(A \cup B) - P(A) - P(B)$

   (j) $1 - P(A^C \cup B^C)$

6. Seven balls are tossed randomly into three baskets. Find the probability that one basket contains exactly five balls.

7. The state of California has 53 Congressmen, of which 33 are Democrats and 20 are Republicans. Suppose we take a random sample of five California Congressmen. What is the probability of each of the following?

   (a) There is exactly one Republican in the sample.

   (b) There are two or more Democrats in the sample.

8. Suppose a man goes into a sub shop. He plans to buy a sub with any one of six meats on any one of four types of bread. He can also have any combination of twelve different additions (such as vegetables and condiments.) How many different subs could he order?

9. The judges of a dog show will award one Best in Show, Second Prize, Third Prize, and two Honorable mentions. If there are 18 contestants, how many distinct ways can they award these prizes?

10. If we select a random permutation of the letters in "TENNESSEE", what is the probability of each of the following?

    (a) All the E's are together?

    (b) Both of the S's come before the T?

    (c) Either of the S's comes before the T?

    (d) The N's are in the first and last places?

11. A driver's license ID number is ten digits long, with the format XXXX-XX-XXXX. Each of the first four digits must be even, the next two digits cannot be 00, and the last four digits cannot contain any 9's. How many different driver's license ID's are possible?

12. There are two crates with fruit in them. The first one has 2 apples and 3 pears. The second one has 30 apples and 40 pears. Since the first crate is lighter, you decide on the following randomization scheme - you will flip a coin twice and select the second crate only if both flips are tails. Otherwise, you will select the first crate. You select one piece at random from the chosen crate.

    (a) Find the probability that you will choose a pear.

    (b) Given that you selected a pear, what is the probability that you chose Crate 1?

    (c) Given that you selected an apple, what is the probability that you chose Crate 1?

    (d) Why does the sum of the answers in (b) and (c) not equal 1?

13. Suppose we choose one child a random from a given population. For this population, the probability that the child is a fan of Harry Potter is 65%. The probability that the child is a female fan of Harry Potter is 29%. What is the (conditional) probability that the child is female given that they are a Harry Potter fan?

14. Team A needs to win one of its final two games to make the playoffs. The probability Team A will beat its first opponent is 0.72. If it wins this game, the probability it will win the second game is 0.44 (since the starters will rest.) If Team A loses the first game, it will win the second with probability 0.87.

    (a) Define all the conditional probabilities given in the problem.

    (b) What is the probability A makes the playoffs?

    (c) What is the probability that A wins both games?

15. Two fair eight-sided dice are rolled. (Each is numbered from 1 to 8.)

    (a) What is the probability that neither one is 1 or 2?

    (b) If the two faces are different, what is the probability that neither one is either 1 or 2?

16. Suppose I have three matchboxes. Some of the matches are wet and do not light. All of the dry matches will light. In the first box, there is one wet match out of 10. In the second, there are three wet matches out of 20. The third box contains only wet matches. I will select one matchbox and then randomly draw one match from the chosen box.

    (a) What is probability that the match will light?

    (b) Given that the match did light, what is the probability it came from the first box? The second box? The third box?

(c) Suppose instead you draw two matches at random from the chosen matchbox. If only one of them lit, what is the probability that you selected the first box?.

17. Suppose you are playing Scrabble and are about to draw two letters from the bag. You have been keeping track of the letters drawn and know that the only tiles remaining in the bag are 3 E's, 2 A's, 1 N, and 3 letters you can't use. What is the probability that you get each of the following?

    (a) One E and one other letter (i.e. not an E).
    (b) One A and one E.
    (c) Two unusable letters.

18. English and American spellings are *rigour* and *rigor* respectively. A man staying at a Parisian hotel writes this word, and a letter taken at random from his spelling is found to be a vowel. If 70% of the English-speaking men at the hotel are English and 30% are American, what is the probability that the writer is an American?

19. A boss is determining how to divide bonus money among three employees. He has a total of $12,000 to allocate and can only award bonuses in multiples of $1,000. If he has to use all of the money and has to give at least $1,000 to each person, how many different ways can he do this?

20. The kids of a second-grade classroom are asked if they prefer chocolate or strawberry ice cream. Every child answered. There were 10 boys and 8 girls who preferred chocolate, and there were 6 boys and 8 girls who preferred strawberry. Are the gender of child and ice cream preference variables independent? Why or why not?