Lecture 16
Relationship between Discrete Distributions and Nested Problems

STAT 225 Introduction to Probability Models
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Agenda

1 Relationship between Discrete Distributions

2 Nested Problems
Choosing a Distribution

Nested Problems
In some of our examples, we will use a distribution to calculate a probability, then use that probability as the parameter in a new distribution.
Example 40

In a jar there are 200,000,000 coins, 5,000,000 of which are quarters. You select 50 coins from the jar randomly and without replacement. Let \( X \) be the number of quarters in your sample.

- What is the distribution of \( X \)?
- Find the probability that \( X \) is 2
- Is there an approximate distribution for \( X \), why or why not?

Example 40 cont'd

Solution.
Example 41

Nick plays a game with his friend Eric. Eric bets $1 every hand (5 cards). If he gets a full house, he wins $500 (on top of keeping his bet of $1); otherwise, he loses the $1 to Nick. Suppose in an afternoon of gambling, Nick and Eric play this game 500 times. Let $E$ denote the number of hands that Eric wins in this particular afternoon.

1. Name the distribution and parameter(s) for $E$
2. Find the probability that $E$ is at least 3
3. Is an approximation appropriate for $E$? Why or why not?
4. If an approximation is appropriate, find $P(E \geq 3)$

Example 41 cont'd

Solution.
Example 42

The wonderful candy shop, Albanese Candy Outlet, makes chocolate chip cookies as part of their production line. Chocolate chips in the cookies are randomly and independently distributed with an average of 12 chocolate chips per cookie. You and 9 of your friends decide to make a trip to Albanese Candy Outlet. Each of you buys one chocolate chip cookie.

1. What is the probability that your cookie contains between 10 and 15 chocolate chips inclusive?
2. What is the probability that 5 or 6 people in your group have cookies with between 10 and 15 chocolate chips inclusive?
3. While examining your cookies (one-by-one), what is the probability that it takes at least 4 cookies to find the first one with between 10 and 15 chocolate chips inclusive?
4. Suppose you and your 9 friends were to go repeatedly to Albanese Candy Outlet. What is the probability that it takes until your sixth trip so that 5 or 6 people in your group have 12 or 13 chocolate chips in their cookie?

Example 42 cont’d

Solution.
Example 42 cont'd

Solution.

Example 43

An urn contains 6 red balls, 6 green balls, and 3 purple balls. You randomly reach in and pull out 4 balls.

1. Assume sampling is done with replacement. What is the probability that you draw at least 2 purple balls?
2. Assume sampling is done without replacement. What is the probability that you draw at least 2 purple balls?
3. Assume sampling is done with replacement. What is the probability that it takes you until your tenth sample to get a sample with at least 2 purple balls?
Example 43 cont'd

Solution.