Assignment 6
(Deadline: 03/07/2014)

1. (Ex8.3) Suppose that we observe \( m \) iid Bernoulli(\( \theta \)) random variables, denoted by \( Y_1, \ldots, Y_m \).

Show that the LRT of \( H_0 : \theta \leq \theta_0 \) versus \( H_1 : \theta > \theta_0 \) will reject \( H_0 \) if \( \sum_{i=1}^{m} Y_i > b \).

2. (Ex8.5) A random sample, \( X_1, \ldots, X_n \), is drawn from a Pareto population with pdf

\[
f(x|\theta, \nu) = \frac{\theta \nu}{x^{\theta+1}} I_{[\nu, \infty)}(x), \quad \theta > 0, \nu > 0.
\]

(a) Find the MLEs of \( \theta \) and \( \nu \).

(b) Show that the LRT of \( H_0 : \theta = 1, \nu \) unknown versus \( H_1 : \theta \neq 1, \nu \) unknown,

has critical region of the form \( \{x : T(x) \leq c_1 \text{ or } T(x) \geq c_2 \} \), where \( 0 < c_1 < c_2 \) and

\[
T = \log \left[ \frac{\prod_{i=1}^{n} X_i}{(\min_i X_i)^n} \right].
\]

3. (Ex8.6) Suppose that we have two independent random samples: \( X_1, \ldots, X_n \) are exponential(\( \theta \)), and \( Y_1, \ldots, Y_m \) are exponential(\( \mu \)).

(a) Find the LRT of \( H_0 : \theta = \mu \) versus \( H_1 : \theta \neq \mu \).

(b) Show that the test in part (a) can be based on the statistic

\[
T = \frac{\sum X_i}{\sum X_i + \sum Y_i}.
\]

4. (Ex8.9) Suppose \( Y_1, \ldots, Y_n \) are independent with pdfs \( \lambda_i e^{-\lambda_i y_i} \), and we want to test \( H_0 : \lambda_1 = \cdots = \lambda_n \) versus \( H_1 : \lambda_i \) are not all equal. Show that the LRT statistic is given by \( (\bar{Y})^{-n}/(\prod_i Y_i)^{-1} \) and hence deduce the arithmetic-geometric mean inequality.

5. (Ex8.10) Let \( X_1, \ldots, X_n \) be iid Poisson(\( \lambda \)), and let \( \lambda \) have a gamma(\( \alpha, \beta \)) distribution, the conjugate family for the Poisson. Now consider a Bayesian test of \( H_0 : \lambda \leq \lambda_0 \) versus \( H_1 : \lambda > \lambda_0 \). Calculate the expressions for the posterior probabilities of \( H_0 \) and \( H_1 \).